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THE UNIVERSITY OF ALBERTA
A REMOTE ERROR SIGNAL GENERATOR
FOR AN AUTOMATICALLY CONTROLLED FIELD TRACTOR

by

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A THESIS
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The undersigned certify that they have read and recommend to the Faculty of Graduate Studies for acceptance, a thesis entitled "A Remote Error Signal Generator for an Automatically Controlled Field Tractor" submitted by George Ronald Liivam in partial fulfilment of the requirements for the degree of Master of Science.

ABSTRACT

The principal objective of this thesis project was to design a practical remote error signal generator that could be incorporated into an automatic control system for a farm tractor. The practicality of the system was related to the economic situation of the day. Since this thesis was prepared at a time when prices were rising rapidly due to inflationary tendencies in the economy, and some prices were dropping fast because of technological advances in the electronics industry, the estimated price of the error signal generator had to be put in a range of twelve to eighteen thousand dollars.

The detection system used a method of triangulation to locate the tractor. Infrared detectors were used to indicate when the tractor was in the detector field. The angular position of the detector was related through a gear train to the position of the shaft of a digital shaft position encoder. The output of the encoder was a digital indication of the angular position of the tractor relative to the base line with an axis of rotation passing vertically through the detector. A digital computer was used to calculate the cotangents of the angles and to perform logical operations on the information obtained.

The control system was found to be stable for a range of velocity commonly used in field operations. With greater demand put on the detection system, higher velocities could be attained.

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CHAPTER I

INTRODUCTION AND THE PROBLEM

A current review of automation in agricultural field operations, shows that various forms of mechanization are being developed to compensate for the shortage of labor. The emphasis on the problem will undoubtedly lead to the development of automatic control systems because of the significant decrease in human involvement possible. This statement is not implying that automatic control systems do not already exist in agricultural operations but that the sphere of application is limited to specific minor operations. Major areas exist in which measurement and control systems can be expected to play an increasing role in the future.

Manufacturers of instrumentation and control equipment are aware of the vast market for their services in agricultural production and are anxious to exploit this situation. However there is no immediate revolution to be expected as a result of this interest. When problems of the environment and of the agricultural production have been overcome, there still remain considerations of price and likely demand which often discourage the development of commercial equipment. Nevertheless, the continuance

of present trends in agriculture and in the electronics industry must, in time, favor further developments. The farmer's growing expenditure on mobile and static equipment is an obvious trend of this kind. Ensuing economy in manpower and effort, improvement in production equality and quantity, and further use of favorable weather and soil conditions justify considerable outlay. In electronics an encouraging sign is the growing availability of the packaged solid state units and integrated circuits for measurement and control systems. By employing these in agricultural applications, maximum price advantage can be attained in addition to simplified construction and replacement.

1.2 LOCATION AND CONTROL OF FIELD TRACTORS

The general purpose tractor is the main source of power in North American agricultural field crops. Currently these are not designed to give the driver a reasonable degree of comfort or safety. For this and the reasons given in Section 1.1, engineers in various parts of the world have been exploring ways to dispense with the driver for some types of field operations. Several alternate approaches have been taken in attempts to limit the level of human intervention in the control of the tractor.

The most widely explored method is the use of radio control^{1,2}, the tractor responding to commands which

operate tuned relays linked by electrical or electrod-hydraulic devices to the controls. Immediate applications are for operating the tractor in areas where some form of hazard exists, and control of more than one tractor by a single operator, i.e., tandem cultivation with the operator on one tractor and the slave tractor following closely³. For more remote control, closed circuit television is necessary.

The radio control system may in these ways increase output or make the operator's life more pleasant, but it does not offer the best approach to a more ambitious project -a fully automatic tractor guidance system. The attraction of such a system is that it may conserve labor by freeing the driver -at least in part- from the more routine operations and enable him to carry on with work requiring a greater degree of human intervention. The regular shape and topography of the fields in Western Canada, permit a number of possible guidance systems.

A direct line of sight system employing the laser beam is described by Williamson⁴. The drift limit with the laser detector is 5/8 inch in 2 miles. A gas laser is mounted parallel to the telescope of a surveyor's transit. The laser's narrow continuous beam is aligned by the transit to fall on the center of a rectangular array of photocells mounted at front and back of the machine. The photocells are tied in with

an identical array of lamps on a panel in front of the driver. Thus the illuminated photocells light equivalent lamps on the panel, giving the operator an identification of how closely he is tracking the laser beam.

At the NIAE⁵, a system employing ground markers, has been investigated. A working model has been constructed on a tractor with an infinitely variable hydrostatic transmission. A similar system using furrow following links is described in Farm Mechanization⁶. Another version of ground marker is the one using a leader cable. This was developed by Morgan⁷ at Reading University. Finn-Kelcy⁸ and Owen^{8a} recently presented a paper on an automatic system of programmed tractor control using the same technique as Morgan.

Another dead-reckoning control system which was fitted to a tractor was described by Gilmour⁹. Richey¹⁰ developed a system that operated on the principle of plant feelers. Although this system did not eliminate human intervention, it definitely decreased the degree of human concentration necessary.

MacHardy¹¹ described an automatic guidance system for a farm tractor. The system would keep the tractor on a course parallel to a given reference line, with only three level (-, +, 0) position information. He proposed a method for determining the position of the tractor, but did

not research this aspect of the project. Shukla¹², at Missouri State University, is working on a control system that is based on somewhat the same principle, but to date there is no record of a publication.

1.3 OBJECTIVES OF THIS THESIS

The objectives of this thesis are as follow:

- a) To describe a practical system for determining the position of the tractor in an automatic control system for a farm tractor.
- b) To evaluate the precision obtainable with the system.
- c) To estimate the cost of the hardware involved in the system.

1.4 OUTLINE OF THE PROJECT

A synopsis of the automatic guidance system to which the error detection system is to be applied will follow in the next chapter. The characteristics required in the error generator will then follow. The equipment will be described generally and mathematically in ensuing sections. The last sections will be an evaluation of the response of the complete system, to a set of initial conditions.

CHAPTER II

SYNOPSIS OF THE GUIDANCE SYSTEM

A tractor guidance principle that served as a starting point for the research leading to this thesis, is described in a paper published by MacHardy¹¹.

MacHardy simulated the tractor control system on an analog computer, and built a working tractor model.

In the paper mentioned above, MacHardy also proposed a method for locating the tractor by means of infrared detectors. The system for locating the tractor was not investigated however. The extension of the project to include the system for locating the tractor is the prime object of this thesis.

2.2 REVIEW OF THE GUIDANCE SYSTEM THEORY

A review of the control system referred to (11,13) follows.

MacHardy proposed to guide a tractor along the path rr' , Fig. 2.1, which is parallel to the reference line AB.

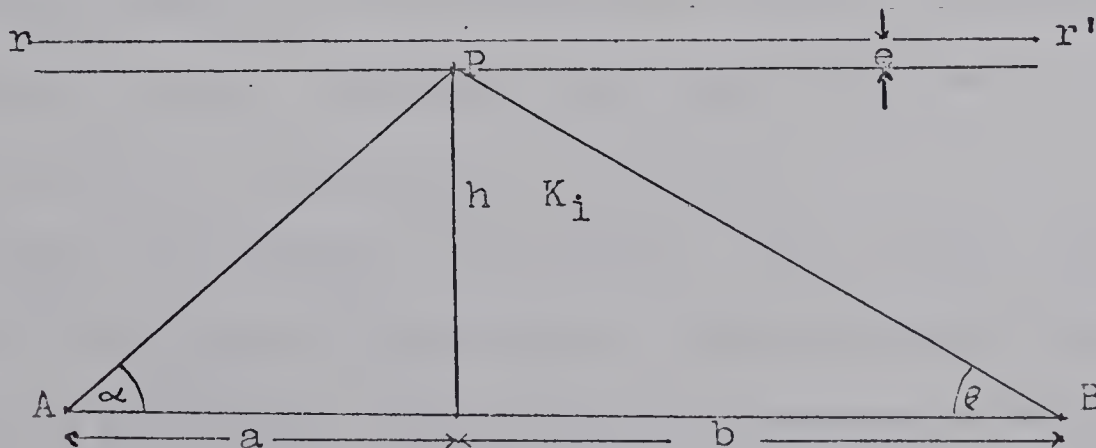


FIGURE 2.1 LOCATING THE POINT P BY TRIANGULATION

The distance between the observation points A and B is known. P is the actual position of the tractor; therefore it is a distance h above the baseline AB.

Since the sum of $\cot\alpha$ and $\cot\beta$ is constant

$$a/h + b/h = AB/h$$

for all points on a path parallel to AB,

$$\frac{AB}{\cot\alpha + \cot\beta} = h \quad 2.2.1$$

Should the distance h be different from any desired tractor location K_i , an error can be measured as

$$e = \left[AB / (\cot\alpha + \cot\beta) \right] - K_i \quad 2.2.2$$

MacHardy proposed that the error e should have three levels, (+, -, 0) depending on the location of the tractor.

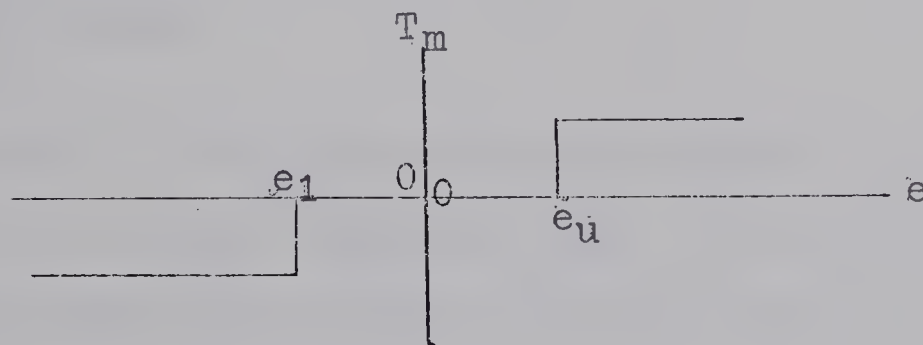


FIGURE 2.2 ERROR SIGNAL OUTPUT AS A FUNCTION OF THE DISPLACEMENT y

Figure 2.2 illustrates the error signal as a nonlinear function of the lateral tractor displacement e where e_u and e_l represent the upper and lower limits within which it is proposed to control the tractor. This signal determines the tractor front wheel angular position as turned to the left, straight ahead and turned to the right. The tractor response to a turning

signal may be described as follows:

Let T_m be the angular position of the front wheels in radians per foot, and let T_a be the angular position of the tractor with respect to the baseline in radians.

Then

$$T_a = v \int_0^t T_m dt \quad 2.2.3$$

where v is the forward velocity of the tractor in feet per second. Let y be the lateral distance from the reference line. Then

$$y = v \int_0^t \sin T_a dt \quad 2.2.4$$

For $T_a < 0.20$, $\sin T_a \approx T_a < .01 \sin(T_a)$;

hence for small angles, let $\sin(T_a) = T_a$.

Then

$$y = v^2 \int_0^t \int_0^t T_m dt dt \quad 2.2.5$$

It was proposed to damp the steering system by using a gyrocompass in a minor feedback loop. The gyrocompass would return some part of the velocity with which the lateral position of the tractor was changing. If the damping is set to b , then the equation developed by MacHardy, 2.2.5, describing the steering control system, becomes

$$y = T_m v^2 \int_0^t e^{-bt} dt \quad 2.2.6$$

2.3 GENERAL ANALYSIS OF THE INTEGRO-DIFFERENTIAL EQUATION

If the open loop transfer function of the guidance system, Eq. 2.2.6, is expressed in the Laplace domain; it

becomes

$$G(S) = \frac{C(S)}{E(S)} = \frac{T_m v^2}{s(s+1)} \quad 2.3.1$$

(8)

in which T_m is a coefficient that can have only one of three discrete values at a time. The error signal

$$E(S) = R(S) - C(S) \quad 2.3.2$$

Eliminating $E(S)$ from Eq. 2.3.1 and letting $K = T_m v^2$ results in

$$(S(S+1))C(S) + KC(S) = KR(S) \quad 2.3.3$$

Taking the inverse Laplace transform, Eq 2.3.3 becomes

$$\frac{d^2 c(t)}{dt^2} + \frac{dc(t)}{dt} + Kc(t) = Kr(t) \quad 2.3.4$$

If $r(t)$ is zero, Eq. 2.3.4 becomes

$$\frac{d^2 c(t)}{dt^2} + \frac{dc(t)}{dt} + Kc(t) = 0 \quad 2.3.4$$

Substitute $x=c(t)$ and $y=\dot{x}$ in Eq. 2.3.5. Then the normalized equation becomes

$$\dot{y} = -y - Kx \quad 2.3.6$$

Divide Eq. 2.3.6 by $y = \dot{x}$; then

$$\frac{\dot{y}}{\dot{x}} = -1 - \frac{Kx}{y} \quad 2.3.7$$

The last equation presents an analytical expression for the slope of a trajectory in the (x, \dot{x}) plane. For an input $r(t) = 0$, there exists one trajectory in the phase plane for each set of initial conditions $x(0)$, and $\dot{x}(0)$.

Absence of any forcing function in Eq. 2.3.7 implies that all system excitation exists in the form of an initial lateral dislocation and angular position of the tractor. In the phase plane the trajectory starts at the point described by the initial conditions. Using isoclines (loci in the phase plane that have constant slope), the plot of a trajectory in the phase plane, Fig. 2.3 is obtained.

Table I contains information on the slope of the trajectory in the region of the isoclines in the phase plane. These regions of constant slope, facilitate plotting the complete response of the system for a set of initial conditions. The initial conditions of the trajectory, Fig. 2.3, are (4.0, 0); in which the 4 represents a lateral displacement of the tractor of 4 feet and the 0 indicates that the lateral position of the tractor is not changing.

TABLE I

TABLE OF VALUES SATISFYING $dy/dx = -1 - Kx/y$

dy/dx	$y = f(x)$
-6	1.0x
-5	1.3x
-4	4.8x
0	-5.9x
1	-2.8x
2	-1.8x
3	-1.4x
5	-0.9x
6	-0.7x

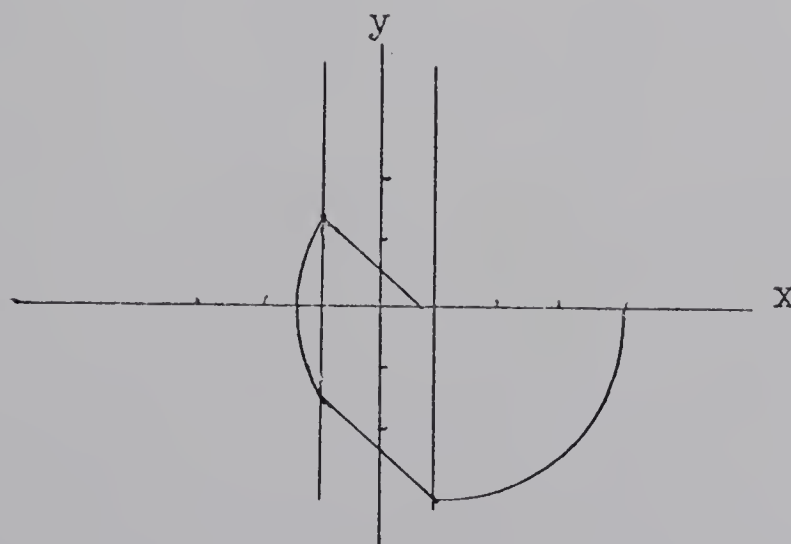


FIGURE 2.3 PHASE PLANE PLOT OF THE GUIDANCE SYSTEM RESPONSE TO INITIAL CONDITIONS I.c.(4.0,0)

The phase plane trajectory, Fig. 2.3, demonstrates graphically that the system described by MacHardy was stable for some initial conditions.

CHAPTER III

DETECTION SYSTEM COMPONENTS

Whereas the choice of components for the guidance system developed as alternatives were considered, (and often rejected), it is felt that there should be a brief description of the system's components as finally decided upon, before discussing the reasoning that lead to the particular choices.

Each infrared detector is scanned horizontally through a small arc about the target position. The angular position of each detector is measured by a shaft position encoder with a digital output. The encoder output is connected to interfacing equipment and a PDP-8 digital computer. The computer is programmed to compute the cotangents of the angles locating the target from both detectors, then to sum these cotangents and compute the tractor lateral location as indicated in Section 2.3. The actual lateral position is compared with the programmed position, and if these are unequal, an error signal is radioed to the tractor steering mechanism. Figure 3.1 shows a simplified diagram of this system and its components.

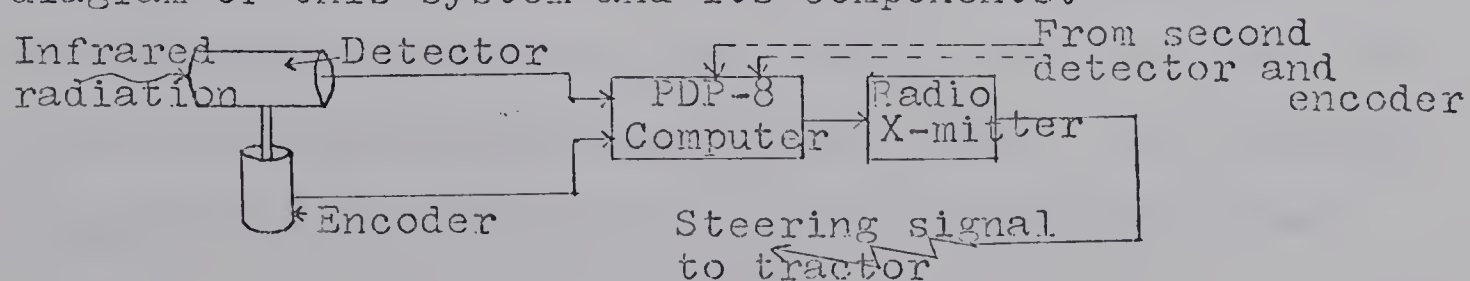


FIGURE 3.1 ERROR DETECTION SYSTEM

3.2 GENERAL DESCRIPTION OF THE DETECTOR

Infrared detectors are sensing devices that detect objects whose surface temperatures are effectively different from that of their surroundings. The first concentrated interest in their development was evident during the first world war when the German scientists began exploring their use in military applications. During the second world war, the interest again increased and devices for following equipment movements at night were developed and improved.

Today the military applications of detectors still outnumber the industrial and medical applications. Missile seekers and homers (used in Sidewinder air to air missiles)¹⁵ use infrared detectors to find and lock on hot areas of the intended target. Other military applications are fire control, bomber defence, and the ballistic missile detection. The two main reasons for their popularity in military applications are (1) the passive nature of their detection system and (2) the increased angular refinement. Unlike radar, they usually do not emit a signal. This renders them undetectable. Active systems¹⁶ do exist; however they are not widely used.

Peaceful applications are widely diversified. One detector developed by the NRC¹⁷ is used to detect hot spots in high voltage transmission lines. The forestry departments use detectors to locate accurately, the

fires shrouded by heavy smoke, thus increasing the effectiveness of the fire fighting equipment. The detector that was developed by the NRC, is the one after which the detector that was used in the experimental work leading to this thesis, was designed.

There are many more applications and a few other types of detectors in use; however mention has been restricted to those applications that are closely related to the one used in this thesis.

The active components of the Bolometer type infrared detector consist of thin resistive elements exhibiting a large change of resistance with temperature, connected in a bridge circuit as in Fig. 3.2.

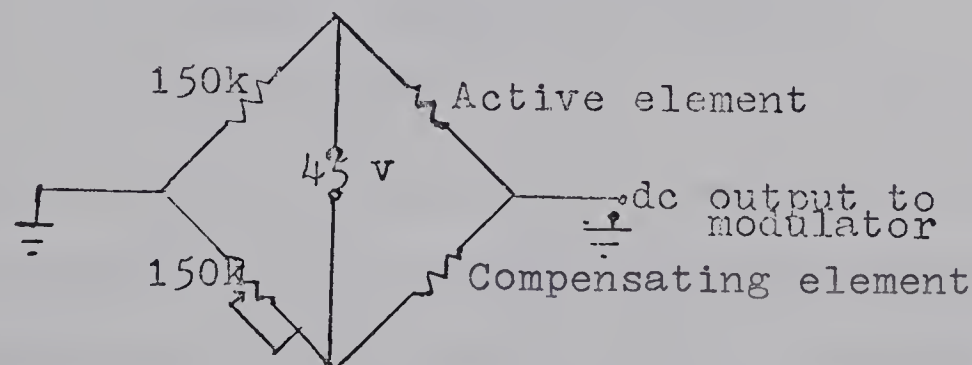


FIGURE 3.2 BOLOMETER BRIDGE CIRCUIT ARRANGEMENT

The sensing element in the detector, is a semiconductor film of a mixture of manganese, nickel and cobalt. These thermistors have large negative temperature coefficients. A rapid response is obtained by attaching the "flakes" to good heat conducting thermal sinks.

The bolometer bridge circuit, Fig. 3.2, comprises two identical thermister elements mounted on the same

base to effect compensation for changes in ambient temperature. One element, the active receiver, is exposed to infrared rays while the other is shielded.

3.3 INPUT-OUTPUT RELATIONSHIP OF THE DETECTOR

Fig. 3.2 (a) shows the functional units of the electronic system.

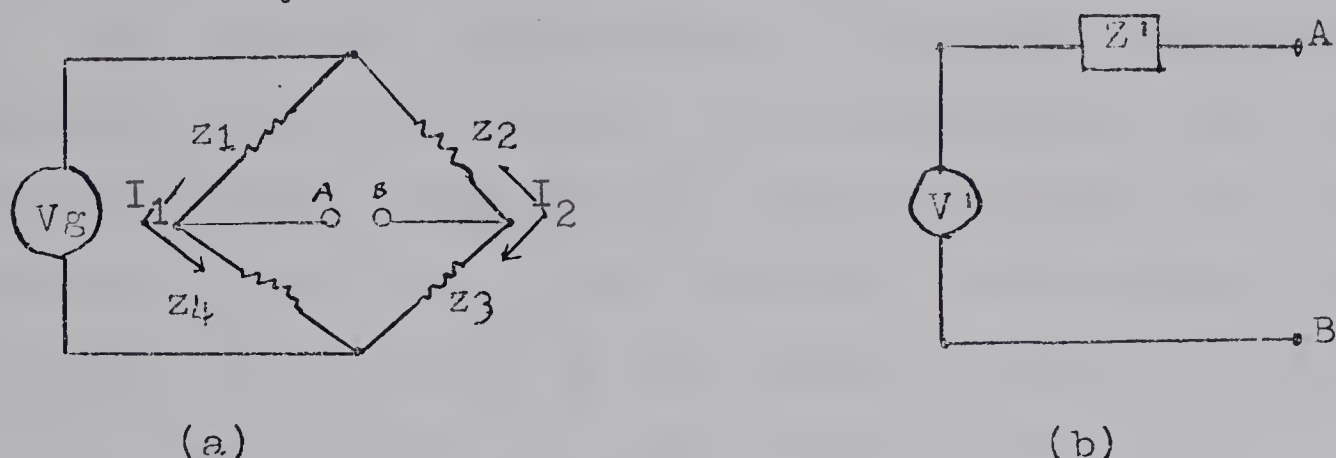


FIGURE 3.3 (a) FUNCTIONAL UNITS ARRANGED IN A BRIDGE CIRCUIT; (b) THEVENIN EQUIVALENT CIRCUIT.

When the voltage source is set to zero, the equivalent impedance seen at terminals AB consists of the parallel combination of z_2 and z_3 , in series with the parallel combination of z_1 and z_4 . Thus in equation form

$$Z' = \frac{z_1 z_4}{z_1 + z_4} + \frac{z_2 z_3}{z_2 + z_3} \quad 3.3.1$$

On open circuit the source V_g results in currents I_1 and I_2 as shown in Fig. 3.3,(a).

$$I_1 = \frac{V_g}{z_1 + z_4} \quad \text{and} \quad I_2 = \frac{V_g}{z_2 + z_3} \quad 3.3.2$$

Assuming the potential of A greater than B,

$$V' = V_{AB} = I_1 z_4 - I_2 z_3 \quad 3.3.3$$

$$= \frac{V_g z_4}{z_1 + z_4} - \frac{V_g z_3}{z_2 + z_3} \quad 3.3.4$$

$$V' = V_g \left[\frac{z_2 z_4 - z_1 z_3}{(z_1 + z_4)(z_2 + z_3)} \right] \quad 3.3.2$$

The Thevenin equivalent voltage V' is proportional to the difference $z_2 z_4 - z_1 z_3$. When $z_1 z_3 = z_2 z_4$, the voltage $V' = 0$.

3.4 TIME RESPONSE OF THE DETECTOR

The level of response that the detector reaches is dependent upon two factors: (1) the position of the target in the detector field and (2) the duration that the target remains in the field of the detector. The response of the detector as a function of the target's position in the field of the detector is illustrated in Fig. 3.4.

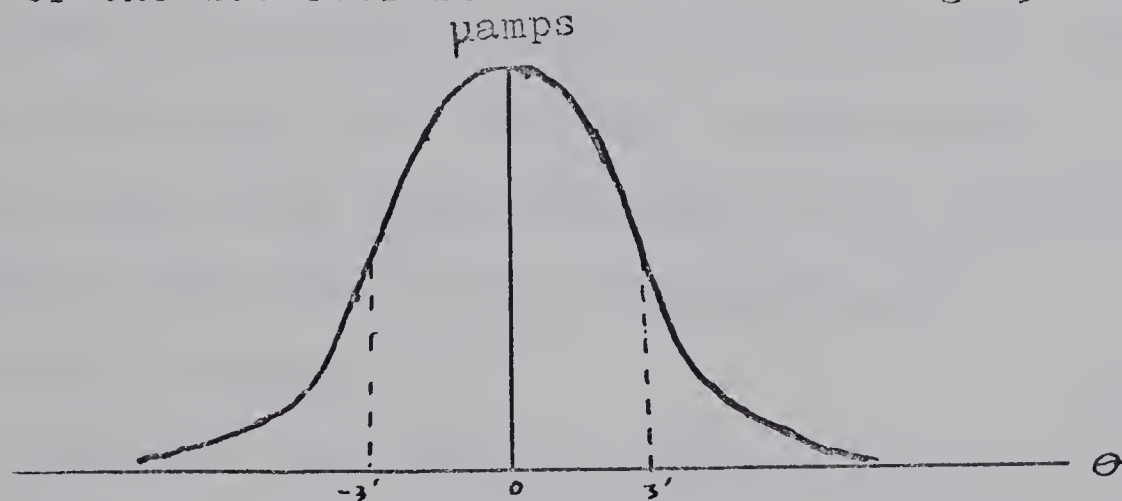


FIGURE 3.4 RESPONSE OF THE DETECTOR AS A FUNCTION OF THE TARGET POSITION IN THE DETECTOR FIELD

The response curve shown in Fig. 3.4 is a reproduction from a figure prepared by personnel at the manufacturing laboratory.¹⁹ The curve can be approximated by Eq. 3.4.1 in which the magnitude is y ; c is a constant due to imbalance, and the zero point on the axis is at the center of the response curve. The width of the detector field is denoted by r and the factor K is

Eq. 3.4.1, is $1/r/2$ for $\theta \leq \pi r/2$ and 0 for $\theta > \pi r/2$, with r expressed in radians.

$$y = c + 2n \int_{-r/2}^{r/2} -\sin K\theta d\theta \quad 3.4.1$$

n is a factor that is itself a function of time. The factor n is very near linear in its magnitude with respect to time where the time does not exceed $2T_c$ where T_c is the time required for the detector to attain a level of response that is 50% of its final response to a particular disturbing temperature change.

$$n = \int_0^b n(t) dt \text{ for } b < 2T_c \quad 3.4.2$$

$$n = n_k \text{ for } b > 2T_c \quad 3.4.3$$

There need be no concern with n in Eq. 3.4.2 if the width of the field w and the frequency ω are such that the detector will be on target for time $t > 2T_c$. Thus the position of the detector may be described by

$$\theta = w/2 \times (\sin \omega t) \quad 3.4.4$$

To obtain the 50% level of response from the detector at a particular angular position with respect to the target source, w and ω may be varied. Thus for example, if it is desirable to have the 50% level of response occur at the moment the tractor is on the center of the detector field, the angular velocity of the detector scan is made such that the response would reach 50% at that time. If the frequency of scan ω is increased, there must be a corresponding decrease in the angular width of the field scanned in order to maintain the 50% response at the same relative position of the tractor in the detector field.

3.5 FIELD OF VIEW AND SENSITIVITY OF THE DETECTOR

The field of view is defined as the angular width by height of the region of sensitivity. The sensitivity is defined as the ability of the detector to distinguish the target radiation from the infrared rays emitted by the background.

Since a tractor could only be located accurately if the detector could distinguish between the exhaust pipe and the cloud of exhaust gases, it was felt necessary to test the detector's ability to distinguish between these sources of heat. Trials with the detector were conducted by the author in 1966. The purpose of the trials was to determine the practicality of using this mode of detection. The direction of focus for the tests was north. The reason being that the highest degree of interference by the background is anticipated to arise from the south side of objects in the northern hemisphere. Another purely mechanical reason for the choice of direction, was to prevent accidental exposure of the lens to direct rays of the sun.

In the trials, a simulated tractor exhaust muffler was used. The simulated muffler was a rusty heating jacket and fire ring from a side arm heater. A variable speed motor with a centrifugal fan, forced air through the heater. Propane was then adjusted until an exhaust gas temperature comparable to that of a tractor under loaded conditions was attained. This meant that there

was an excess of air of approximately 200% of that required for complete combustion. Data to show the effect of the exhaust gases on the detector response, are contained in Table II.

TABLE II

OUTPUT OF THE DETECTOR AMPLIFIER WITH
VAPORS AND HEATER EXPOSED, WITH
HEATER EXPOSED AND WITH JUST
THE VAPORS EXPOSED

Distance yards	Output micro- amps	Attenuator setting	Direction of view	Field width	Background
220	2.5	.3	north	0°12'	trees
220	12.0	1.0	"	0°12'	"
Heater and vapors exposed					
220	2.5	.3	north	0°12'	"
220	20.5	1.0	"	0°12'	"
Heater exposed only. Vapors shielded by a 4x8 piece of $\frac{1}{2}$ in. plywood.					
220	0	.3	"	0°12'	"
220	.5	1.0	"	0°12'	"
Vapors plus $\frac{1}{2}$ in. of the heater were exposed. In a sweep of 360° the variation of the meter was .2pamp					

Before the above results were obtained, it was necessary to prevent fluctuation of the temperature of the detector barrel, especially in the vicinity of the bolometers. The temperature fluctuation was mini-

mized by shielding the barrel from the wind with a layer of insulating material.

The data in Table II indicate that there is an insignificant contribution by the vapors to the total infrared radiation received by the detector. When the vapors were shielded, the plywood was heated sufficiently to cause the total width of the sensitive region to be increased.

TABLE III
SENSITIVITY AT A DISTANCE OF ONE QUARTER MILE

Distance yards	Output micro- amps	Attenuator setting	Direction of view	Field width	Background
440	.5	.3	north	0°8'	trees and cattle
440	4.0	1.0	"	0°8'	Temp 60°
440	2.5	1.0	"	0°8'	Temp 70°

Width of target 0°2'

Heater and vapors were exposed for these and the remaining trials.

Less than 0.5 microampere variation in a sweep of 360°.

Comparing the data of Table III with that of Table II confirms the law that the intensity of infrared rays decreases as the square of the distance. However there is still substantial output from the detector amplifier at a distance of one quarter mile.

In Table IV it is seen that the ability of the detector to distinguish the target under windy conditions was no different from that on still days. Stability of

the detector system was substantially improved by preven-

TABLE IV

SENSITIVITY UNDER WINDY CONDITIONS

Distance yards	Output Micro- amps	Attenuator Setting	
440	Variable	1.0	The detector amplifier response is unstable
440	2.5	1.0	Barrel now protected from the wind by styrofoam insulating material.

ting fluctuation in temperature of the detector barrel, especially in the vicinity of the bolometers.

TABLE V

DETECTOR AMPLIFIER OUTPUT WITH CHANGING DIRECTION

Distance yards	Output micro- amps	Attenuator setting	Direction of view	Position of sun	Tem- pera- ture
440	2.5	1.0	north	south	60°
440	2.5	1.0	east	S60°E	50°
440	4.0	1.0	N45°E	S55°E	52°

The background for the first two trials was trees; for the last trial above, the background was sky.

Note the substantially higher definition of the target with a background of clear sky. The direction of focus relative to the sun has apparently little or no significant effect on the total infrared radiation of the background.

Table VI indicates that the sensitivity of the detector improved substantially in darkness. A surprising observation is that during a period of moderate rain, the

TABLE VI

SENSITIVITY VARIATION WITH DARKNESS

Time	Output micro- amps	Attenuator setting	Direction of view	Background	Weather	
4PM	3.0	1.0	north	barley	rain	70°
8PM	5.0	1.0	"	"	"	64°
12:30AM	6.0	1.0	"	"	no rain	65°
7:45AM	5.0	1.0	"	"	"	68°
2:00PM	4.0	1.0	"	"	"	75°

No variation detectable for 360° sweep at night.

sensitivity was not noticeably decreased.

3.6 TIME CONSTANT OF THE DETECTOR

To check the time constant of the detector, apparatus was arranged as in the block diagram, Fig. 3.5.

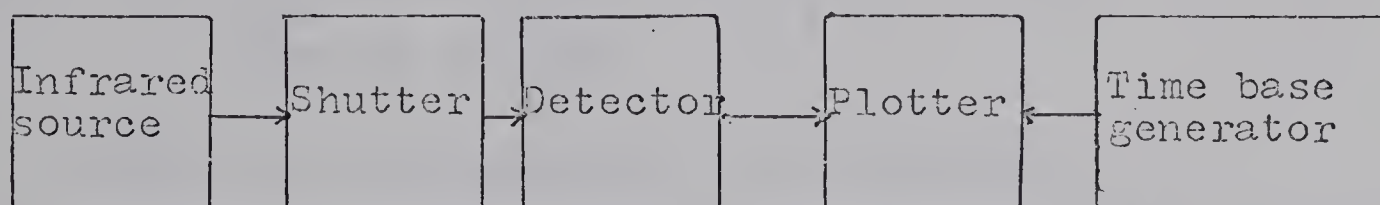


FIGURE 3.5 BLOCK DIAGRAM OF THE APPARATUS USED TO CHECK THE TIME CONSTANT OF THE DETECTOR

The camera lens was removed and the shutter was held open while the lamp was adjusted until a steady output was reached on the plotter. The shutter was then closed to plot the zero position. The time base function was produced by integrating a constant with respect to time and applying this signal to the X-axis of an X-Y plotter.

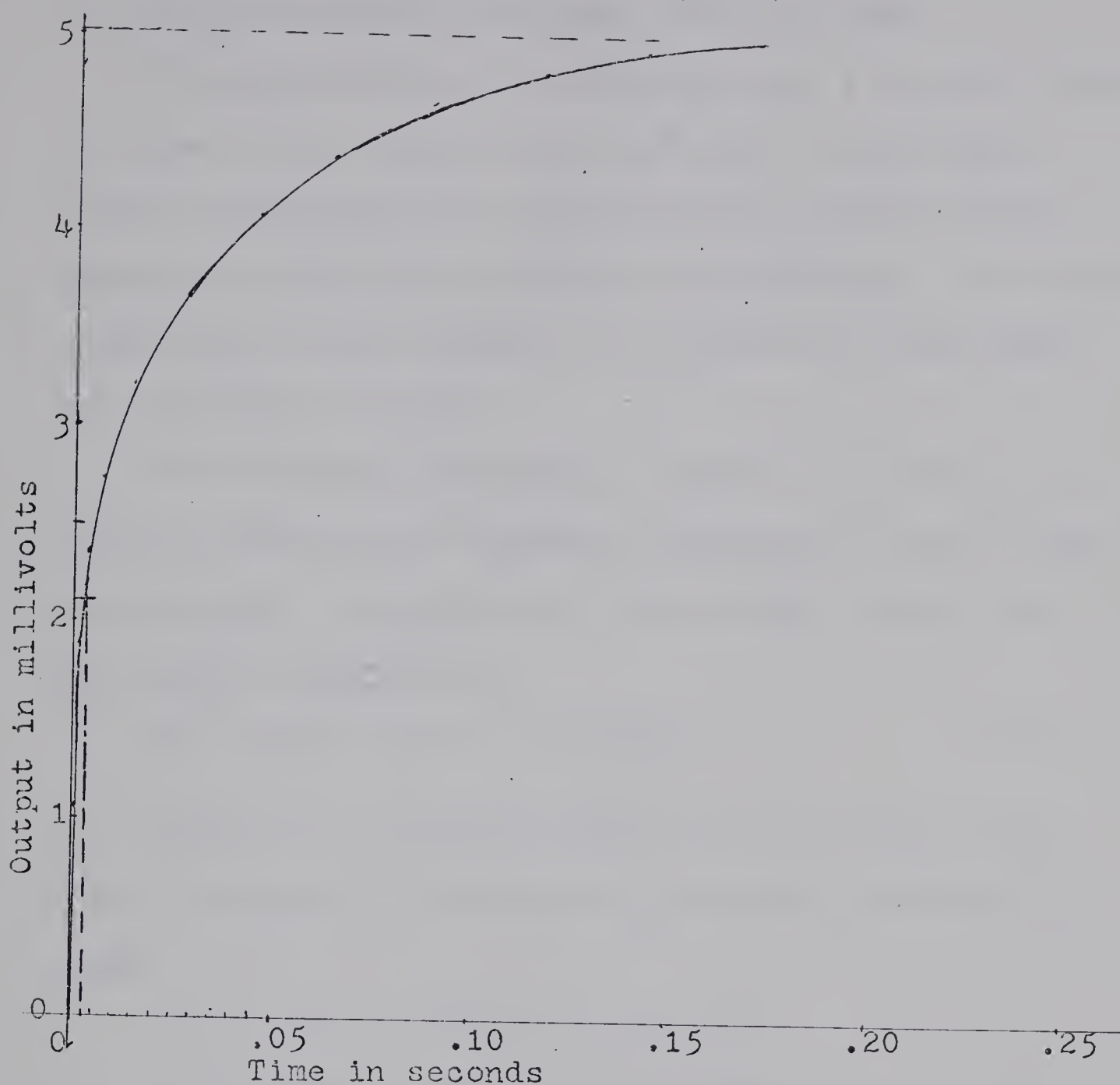


FIGURE 3.6 TIME RESPONSE OF THE DETECTOR

The response curve of the detector, Fig. 3.6, confirms that the time constant of the detector is very close to the manufacturer's specification of 6.5 milliseconds. The 6.5 millisecond level is about 4 millivolts lower than the the 50% level of the steady-state response; however the discrepancy can be charged partly to the plotter.

3.7 SPECIFICATIONS CONTINUED: FIELD OF VIEW

The manufacturer's specifications lists the field of view of the detector as 0.1° wide x 1.5° high. Trials conducted with the instrument by the author, (Section 3.5) confirmed this specification. The angular resolution of the detector is affected by the width of the field of view.

For example, consider a tractor as in Fig. 3.7 with the detectors located at distances A and B from the tractor, the widths of the detector fields are S_a and S_b respectively.

$$S_i = R_i \theta = w_i \quad (i = a \text{ or } b) \quad 3.7.1$$

in which θ is in radians and w is the width of the field of view at a distance R_i from the detector in feet.

$$S_a = 1320 \times .001164 = 1.536 \text{ ft.}$$

$$S_b = 1865 \times .001164 = 2.173 \text{ ft.}$$

The area in which the tractor is located is a parallelogram with sides of 1.536 ft. and 2.173 ft. as shown in Fig. 3.7. The parallelogram has a width of 4.50 feet perpendicular to the intended tractor path rr' . This exceeds the desired limit of plus or minus one foot, thus further refinements in the system are necessary in order to achieve the desired limit. This is referred to again at a later point.

3.8 DETECTOR SCANNING VELOCITY

The time constant of the detector sets an upper limit on the angular velocity of the scanning system. In the absence of any auxiliary scanning devices, the time constant of the detector will not permit a continuous sweep of the field, thus leading to the oscillating drive system that is represented in the block diagram of Fig. 3.8.

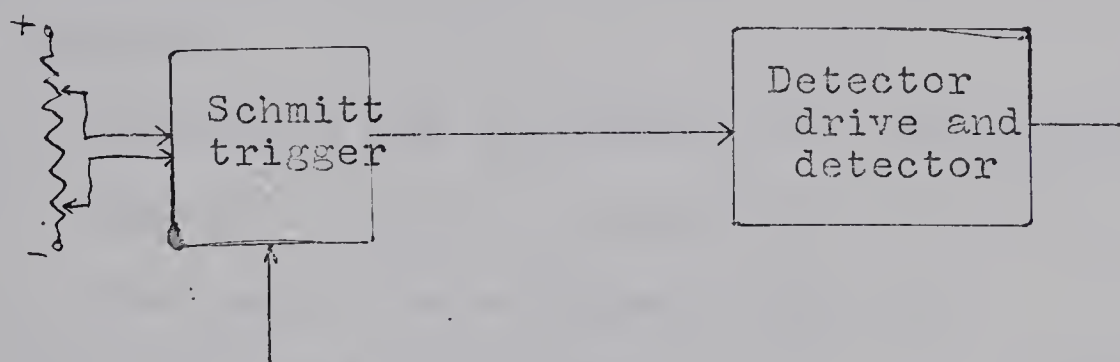


FIGURE 3.8 BLOCK DIAGRAM OF THE DETECTOR DRIVE SYSTEM

In the above system, the feedback for reversing the motor is provided by the detector. The detector feedback is input to a Schmitt trigger and the output from the Schmitt trigger controls the power in the drive circuit. This system ensures that the detector will follow the tractor, while sweeping the region in which the tractor is located with an oscillating motion.

Under normal operating conditions the velocity of scan would be limited to 0.22 radians/second. At this velocity and considering the time delay due to the detector time constant, the detector will produce its pulse when the tractor is at the center of the detector field. Thus a higher degree of precision is achieved.

3.9 LATERAL FREEDOM DUE TO NONSYNCHRONIZATION OF THE DETECTORS

The two detectors in the system result in an added problem.

If the tractor has a velocity of v feet/second and the detectors are out of synchronization by t_0 seconds, the sum of the cotangents will be

$$a/h + (AB - a \pm vt_0)/h \text{ or} \quad 3.9.1$$

$$\cot(a_i) + \cot(b_i \pm t_0 \Delta t) = \frac{AB \pm vt_0}{h} \quad 3.9.2$$

This can also be written as

$$h = \frac{1}{2} \left[\frac{AB + vt_0}{\cot(a_i) + \cot(b_i + \Delta t_0)} - \frac{AB - vt_0}{\cot(a_i) + \cot(b_i - \Delta t_0)} \right] \quad 3.9.3$$

For example, suppose that the tractor is being operated with velocity 8 feet/second at point P, 660 feet from the base line AB in Fig. 3.9, when the first detector (a) registers an output signal. T_0 seconds later detector (b) registers its signal.

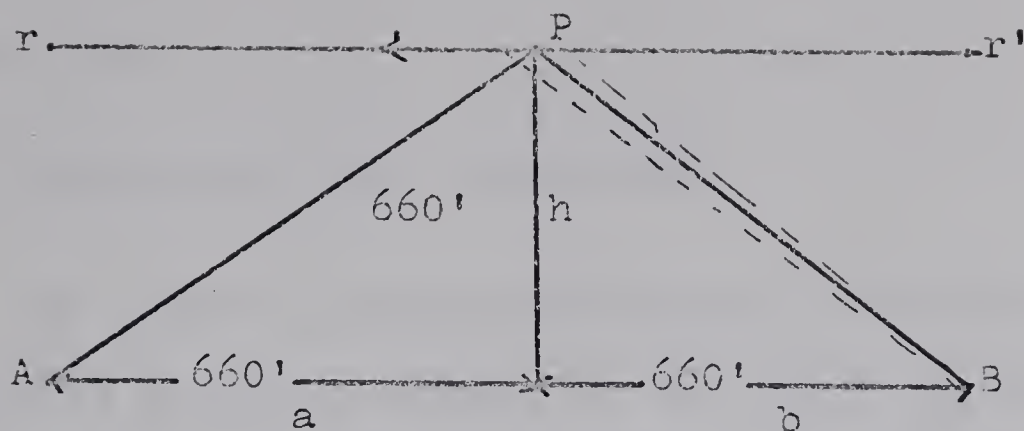


FIGURE 3.9 POSITION OF THE TRACTOR

Then

$$\cot(a) = 660/660 = 1$$

$$\cot(b) = (660 \pm 8)/660$$

Table VIII gives the lateral freedom for various time lags t_0 .

TABLE VIII

LATERAL FREEDOM OF THE TRACTOR DUE TO NON-SYNCHRONIZATION OF THE DETECTORS.

Time period sec.	Sum of Cotangents	AB/ $\cot(a) + \cot(b)$	Lateral freedom feet $\frac{1}{2}(h_{1-t_0} - h_{1+t_0})$
1.0	2.0121212 1.9878787	656.024 664.024	4.024
.75	2.0090909 1.9909090	657.013 663.013	3.013
.50	2.0020202 1.9939393	658.006 662.006	2.006
.25	2.0030303 1.9969696	659.002 661.002	1.002

It is evident from the last column of Table VIII that the lateral freedom decreases linearly with the increase in time lag between the two detectors. Note also that the lag must be less than .25 seconds for the lateral freedom to become less than the control limits set for

the system.

3.10 SAMPLER AND ZERO ORDER HOLD

The train of pulses generated by the detector activates a gating circuit to the input buffer register of the computer. The circuitry is represented by the block diagram of Fig. 3.10.

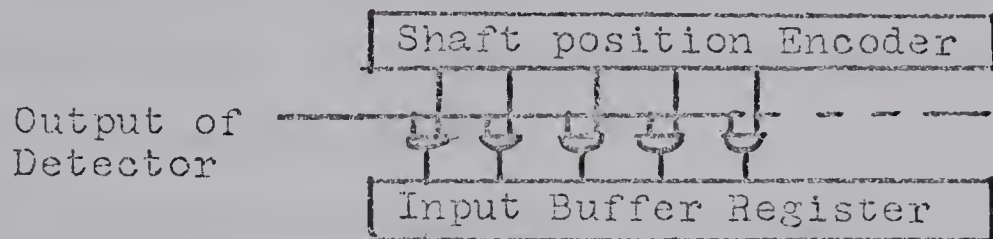


FIGURE 3.10 BLOCK DIAGRAM OF THE INPUT CIRCUITRY

With this arrangement, a degree of frequency modulation can be obtained because the computer program can have dummy operations to make the time for successive samplings equal. The computer will always work with the latest data in the IBR.

Since the buffer register contains information that was collected some time prior to being entered into the computer for processing, and since the processing takes a finite time, there is a definite time delay of b seconds before the signal to the circuitry constituting the guidance system on the tractor, is transmitted. Thus the computer imparts a time delay described in the Laplace domain by e^{-ST} .

For analysis purposes, the time lag caused by the detector, interface equipment, and the computer will be dealt with as a lumped transportation lag e^{-bST} in which

b will be the fraction of the sampling period that the time lag constitutes.

The pulses generated by the detector as its drive system oscillates the detector field across the path of the tractor, are to be used as carrier pulses to activate gates to the IBR in the interface hardware of the computer. The train of pulses therefore, activates the sampler gates.

To simplify the analysis, the sampling circuitry may be replaced by an "ideal" sampler whose output signal $e^*(t)$ contains a train of impulses. This approximation can be justified if the pulse-width of the sampler output is very small when compared with the dominant time constant of the continuous signal $e(t)$, and to the sampling period T .



FIGURE 3.11 SAMPLER

Assume the input is a continuous function $e(t)$ as in Fig. 3.11, the output $e^*(t)$ of the sampler is a train of impulses and is related to $e(t)$ by

$$e^*(t) = e(t)\delta_T(t) \quad 3.10.1$$

where

$$\delta_T(t) = \sum_{n=-\infty}^{\infty} \delta(t-nT) \quad 3.10.2$$

in which $\delta(t-nT)$ represents an impulse of unit area occurring at time $t=nT$. Hence

$$e^*(t) = e(t) \sum_{n=-\infty}^{\infty} \delta(t-nT) = \sum_{n=-\infty}^{\infty} e(nT) \delta(t-nT) \quad 3.10.3$$

Assuming that $e(t)$ is zero for $T < 0$, Eq. 3.10.3 can be written as

$$e^*(t) = \sum_{n=0}^{\infty} e(nT) \delta(t-nT) \quad 3.10.4$$

Taking the Laplace transform of $e^*(t)$ directly from Eq 3.10.4, yields

$$E^*(S) = \mathcal{L}\{e^*(t)\} = \mathcal{L}\left\{\sum_{n=0}^{\infty} e(nT) \delta(t-nT)\right\} \quad 3.10.5$$

$$E^*(S) = \sum_{n=0}^{\infty} e(nT) e^{-nTs} \quad 3.10.6$$

If the Laplace transform of $e(t)$ is a rational function, $E^*(S)$ can be written in a closed form.

i.e., if T is the sampling period and $e(t)=u(t)$, a unit step function,

$$E^*(S) = \sum_{n=0}^{\infty} e^{-nTs} = 1 + e^{-Ts} + e^{-2Ts} + e^{-3Ts} \dots \quad 3.10.7$$

$$= \frac{1}{1-e^{-Ts}} \quad 3.10.8$$

for $|e^{-sT}| < 1$.

If the information input is to be reconstructed, it must be reconstructed from the information contained in the pulse sequence. Using the usual method for generating this approximation, the power series expansion of $e(t)$ in the interval between sampling instants, nT and $(n+1)T$, the following equation results:

$$e_n(t) = e(nT) + e'(nT)(t-nT) + e''(nT)(t-nT)^2 + \dots \quad 3.10.9$$

where

$$e_n(t) = e(t) \text{ for } nT \leq t \leq (n+1)T \quad 3.10.10$$

When only the first term of the power series given by Eq. 3.10.9 is used, the extrapolator is referred to as a zero-order hold. Then Eq 3.10.9 can be written as

$$e_n(t) = e(nT) \quad 3.10.11$$

The impulse response of a zero-order hold circuit is shown in Fig. 3.11

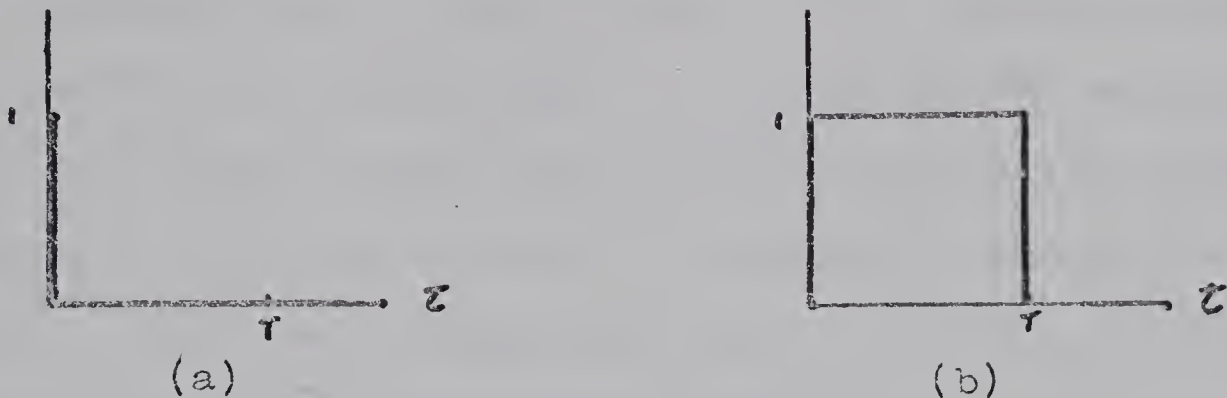


FIGURE 3.12(a) UNIT IMPULSE INPUT TO z-o-h; (b) IMPULSE RESPONSE OF z-o-h TO A UNIT STEP INPUT

The transfer function of the hold circuit is

$$G_{ho}(S) = \frac{1 - e^{-ST}}{S} \quad 3.10.12$$

The combined transfer function of the sampler and z-o-h elements when cascaded is obtained from the following:

$$E^*(S)G_{ho}(S) = \frac{1}{1 - e^{-ST}} \times \frac{1 - e^{-ST}}{S} = \frac{1}{S} \quad 3.10.13$$

Thus the output of the cascaded elements is seen to be a unit step.

A mathematical description of the sampler and the z-o-h have been developed but the input and output signals have not been described. The origin of the input and the destination of the output will be examined in the ensuing sections.

3.11 SHAFT POSITION ENCODER

A shaft position encoder was mentioned earlier without any description of its input-output characteristics other than that its output is a binary indication of the position of the encoder shaft. The Nordan Ketay shaft position encoder provides in binary format, a digital output corresponding to the position of the encoder shaft. The position of the detector is related to the encoder shaft position through a gear train or alternately the angular rotation of the detector can be related to the number of pulses of the least significant bit. The former method is the one used in this control system.

The maximum angular velocity of the shaft position encoder (hereafter referred to as the SPE) is one of the factors that limit the input-output ratio of the gear train between the detector drive system and the SPE. The encoder produces 128 bits/turn and requires 64 turns for full count. The proposed gear train is not restricted to any particular ratio because the conversion of the output to radians is accomplished internally by the computer. Ratios that provide a low angular change of the detector for each bit of the SPE are desirable provided the restrictions imposed by the SPE are not violated.

The only restrictions that are imposed by the SPE are (1) the SPE not be operated at a velocity exceeding

200rpm. and (2) practical gear ratios be used (no half teeth)

Fig. 3.13 is a sketch of the driving circuit and the first three stages of the encoder bit positions.

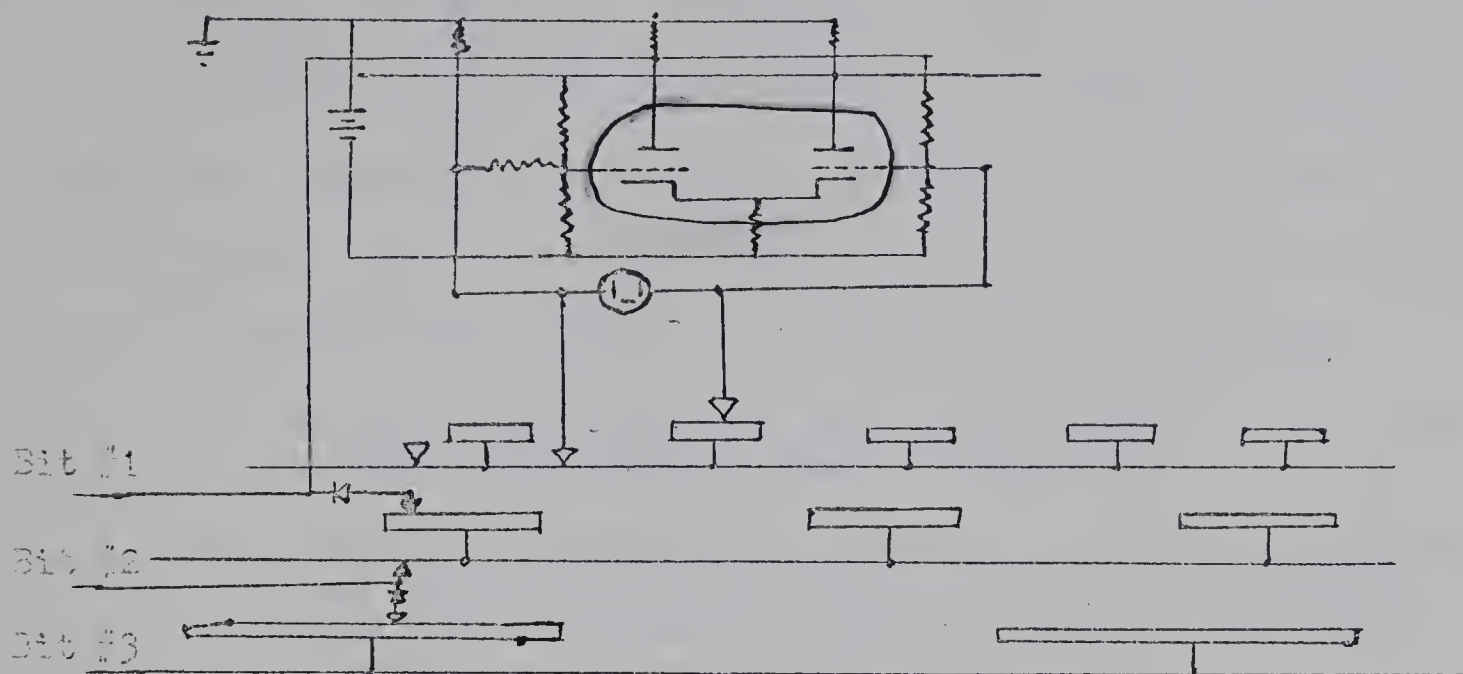


FIGURE 3.13 SPE DRIVING CIRCUIT AND THREE STAGES

TABLE IX

SPE OUTPUT RELATIONSHIP TO SHAFT POSITION

Input position 0 1 2 3 4 5 6 7 8 9 10 11 12 13....

Bit condition

1	0	1	0	1	0	1	0	1	0	1	0	1	0	1...
2	0	0	1	1	0	0	1	1	0	0	1	1	0	0...
3	0	0	0	0	1	1	1	1	0	0	0	0	1	1...
4	0	0	0	0	0	0	0	0	1	1	1	1	1	1...
5	0	0	0	0	0	0	0	0	0	0	0	0	0	0...
.
.
13

The bit condition table, Table IX, confirms that the code is simple binary; thus is compatible with the PDP-8 computer that is referred to in following sections.

Turning at the maximum velocity of 200 rpm the encoder produces $200 \times 128 / 60$ bits per second. With the detector scanning θ°/sec , there will be the equivalent of

$$\theta^\circ \times 60 \times 60 \times .000292 / 200 \times 128 \text{ radians per bit.}$$

The field width θ is referred to again in later sections.

3.12 GENERAL DESCRIPTION OF THE PDP-8 COMPUTER

Analog and digital elements may be used separately or in combination with one another in automatic control systems. Likewise, analog and digital computers can be made compatible. An investigation by the author, revealed the following advantages in using digital computers:

- (1) as much precision as may be required in any particular application may be attained.
- (2) improved reliability because of relative freedom from problems encountered in electromechanical or electronic analog computers, namely precision of components, wear, and stability.
- (3) relative insensitivity to mechanical or electrical noise, which limits the complexity of problems solvable on an analog computer.
- (4) facilities exist for storage of large quantities of data that may be required in complex problems.
- (5) they have the ability to perform logical as well as linear or nonlinear mathematical operations.
- (6) they have flexibility derived from their logical capacity and the fact that their operations are controlled by a program rather than specifically designed or interconnected physical components.

The flow of information through a digital computer system is illustrated by the arrows in the block diagram of Fig. 3.14

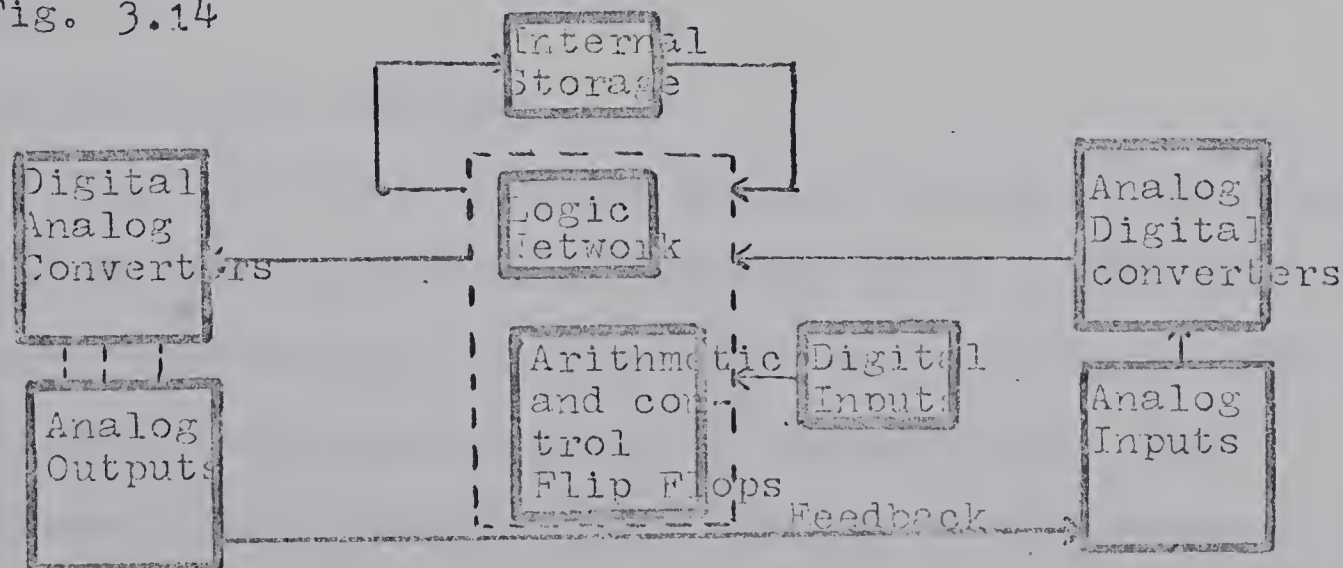


FIGURE 3.14

DATA FLOW IN A DIGITAL COMPUTER

In general purpose computers, time is consumed for information acquisition, obtaining access to data and instructions in storage. Also in the arithmetic operation considerable amount of redundant computing must be done even if the values of the input information change very little from one computation to the next. This is partially offset by the ability of the computer to do computations in large increments, and therefore, the sampling rate in a control system can be varied to obtain optimum stability.

Precision obtainable in a general purpose machine, is governed largely by the numerical method and program selected. To minimize errors, the forms of the equations chosen must be such that they adequately represent the system's characteristics, and are suited to the characteristics of the computer.

The digital computer for which the following work is designed, is the PDP-8 computer manufactured by Digital Equipment Corporation. Minor alterations are necessary for the system to be compatible with other general purpose computers.

3.13 INTERFACE HARDWARE

The PDP-8 is a parallel-transfer machine that distributes and collects data in bytes of up to twelve bits. In the Programmed Data Transfer mode, all programmed data transfers take place through the accumulator. The computer program controls the transfer of information. In-

put data arrive at the AC as pulses received at the interface connectors from bussed outputs of peripheral devices. Gating circuits of the program selected device produce these pulses. Command pulses generated by the device flow to the input/output skip facility (IOS) to sample the condition of the I/O device flags.

3.13.2 Requirements Imposed on Peripheral Equipment

The peripheral equipment must have (1) the ability to sample the select code and when selected, to be capable of producing sequential IOT command pulses (device selector); (2) gating circuits at the output of the transmitting register capable of sampling the information in the output register and of supplying a pulse to the computer input bus when triggered by a command pulse from the DS; and (3) a busy/done flag (flip-flop) and gating circuit which can pulse the computer input/output skip bus upon command from the DS when the flag is set in the binary 1 state to indicate that the device is ready to transfer another byte of information.

3.13.3 Device Selector (DS) (W103)

Bits 3 through 8 of the IOT instruction serve as a device select code. Each DS is assigned a select code and is enabled only when the assigned code is present in the Memory Buffer (MB). When enabled, a DS regenerates IOP pulses as IOT command pulses and transmits these pulses to skip, input gates within the device and to the processor to clear the Accumulator (AC). Fig. 3.15 is a symbolic logic repre-

sentation of the selector.

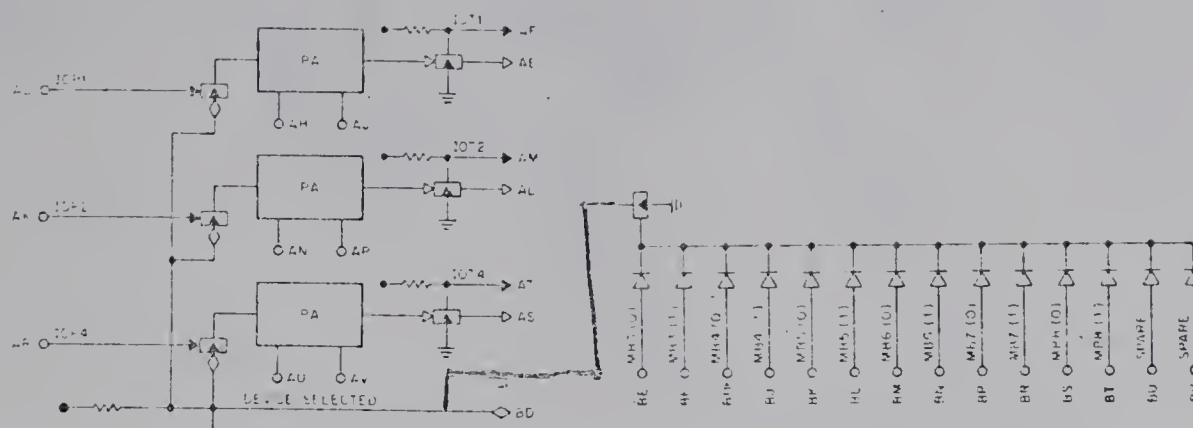
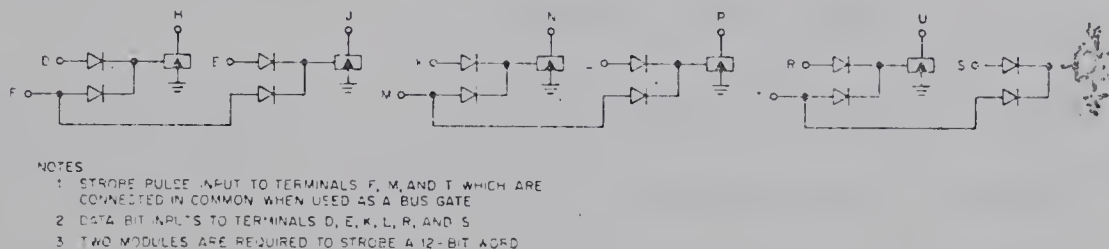


FIGURE 3.15

LOGIC DIAGRAM OF THE DEVICE SELECTOR

3.13.4 R123 Diode Gate

This module contains six dual input NAND gates for negative levels and is primarily used for transferring data into or out of the PDP-8 accumulator. Standard DEC negative levels of 0.4 μ sec. negative pulses such as those from the W103 DS can be used as input signals. Fig. 3.16 is the logical representation of this module.



Diode Gate R123 Logic Circuit

FIGURE 3.16 DIODE GATE

The strobe pulse is input to terminals F, M, and T. Data bit inputs are connected to terminals D, E, K, L, R, and S. Two modules are required to strobe a 12-bit word into the accumulator.

3.13.15 IOP Generator Logic

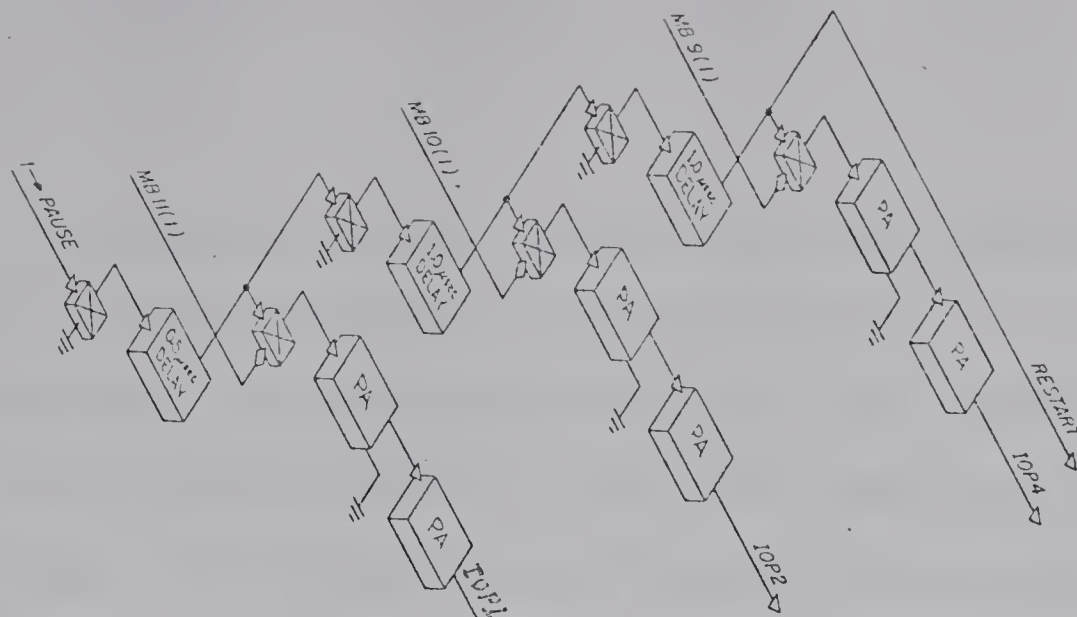


FIGURE 3.17 IOP GENERATOR LOGIC

3.14 COMPUTER PROGRAM

The computer program must instruct the computer to do the following operations:

(1) In logic, the program must distinguish between the two different inputs.

(2) Arithmetically (a) convert the input to operational language;

(b) convert the inputs to radians;

(c) calculate the cotangents;

(d) sum the cotangents.

(3) In logic (a) compare the quotient of $AB \div ct_t$

with the previous quotient for extraneous results; i.e., if $|q_n - q_{n-1}|$ is greater than some limit of validity, wipe out ct_t .

(b) if $|q_n - q_{n-1}|$ is less than the constant of validity then go on with the next comparison;

(c) compare the quotient q_n with the constant k_i (the control constant $r(t)$) $q_n - r(t) = e(t)$.

This error signal must be transmitted to the steering amplifier and converted to analog form if $e(t) < K < -e(t)$.

The operation of selecting an input device was referred to in section 3.13. The DS circuitry is similar to

a sequential multiplexer switch except that the computer program calls for each input at its convenience.

The conversion to operational language is a small operation. The computer works in the two's complement of the binary input. Thus the ones are converted to zeros, the zeros to ones and 1 is added to the result.

The conversion to radians is accomplished by a simple multiplication. The input is multiplied by the conversion factor D referred to earlier on.

The cotangents of the angles can be calculated by solving the power series,

$$\cot(x) = \frac{1}{x} - \frac{x}{3} - \frac{x^3}{45} - \frac{2x^5}{945} - \frac{x^7}{4725} \dots \quad 3.14.1$$

For example, if $\cot(1.0)$ were required, the computer would be programmed to compute

$$\cot(1.0) = 1 - \frac{1}{3} - \frac{1}{45} - \frac{2}{945} - \frac{1}{4725} \dots \quad 3.14.2$$

Since it is desired to know $\cot(1.0)$ to within ± 0.00001 , the series would be terminated after the term that had a value less than 0.00001. Another way of interpreting this result is that the polynomial approximation, Eq. 3.14.2 is sufficiently accurate for our purpose when $x=1$.

Notation: \cot^*x indicates an approximation to the cotangent.

Another method that was considered for obtaining the cotangents of the inputs was by interpolation. The case has been considered where the function $\cot(x)$ is stored in the computer as a subroutine. There are obvious limitations in the method described. The routine might require too much time or too much memory space. In these circumstances

it is convenient to record values of $f(x)$ in the computer memory in the form of a function table which gives the $f(x_i)$ corresponding to a set of specific values of x_i . It appears at first glance that this has great limitations, since only those values of $f(x)$ which correspond to a relatively few values of x_i are in the memory of the computer. However, by means of interpolation, values of $f(x)$ corresponding to x_i 's not listed in the table can be approximated.

This method would certainly be rapid, however the use of storage units is not particularly economical, and the accuracy obtainable would be limited by the length of the word used in the computer. Because of these limitations, a contraction of the power series approximation seemed to offer the better alternative method for obtaining the cotangents. The following algorithm is a contraction of the power series.

Single Precision Cotangent

Algorithm:

- (1) If $x < 0$, use $\cot(x) = -\cot(x)$;
Assume $x \geq 0$.
- (2) Write $x = \pi/4(q) + r$ where q is an integer and $0 < r < \pi/4$. Let $q_0 = q \pmod{4}$.
- (3) If $q_0 = 0$ or 2 (i.e., octant 1 and 3), define $r_0 = r$. If $q_0 = 1$ or 3 (i.e., octant 2 and 4), define $r_0 = \pi/4 - r$.
- (4) Define the case number s as follows; for $\cot(x)$ $s=1$ for $q_0=0$, or $s=0$ for $q_0=1$, or $s=3$ for $q_0=2$, or $s=2$ for $q_0=3$.
- (5) Compute the factor F as follows:

$$F = \frac{1 + 13.946r_0^2 - 313.11}{r_0^2 - 104.46 + 939.33/r_0^2} \quad \text{if } r_0 > 2^{-14} \quad 3.14.3$$

If $r_0 < 2^{-14}$, then $F = 1$

The maximum error of this formula is 10^{-9} .

(6) Now the answer is r_0/F for $s=0$; F/r_0 for $s=1$; $-F/r_0$ for $s=2$; and $-r_0/F$ for $s=3$.

The sum of the cotangents obtained from the above algorithm is arrived at by simple addition. The quotient of the sum divided into the distance between the detectors is the vital byte of information; the steering action is a function of the difference between AB/ct_t and the constant h which represents the desired perpendicular distance between the base line AB and the particular path rr' that the tractor is on. The sum of the cotangents, ct_t , when divided into the distance AB yields a value to compare with the desired perpendicular distance h in Fig. 3.9.

The first check that must be made on this byte of information, is regarding its validity. If $|AB/ct_t - h| > \epsilon$ where ϵ is a constant that represents the bounds of validity, the sum would be discarded. If $|AB/ct_t - h| < \epsilon$ the sum is valid. If $|AB/ct_t - h| < \epsilon$ and $|AB/ct_t - h| > |y|$, where y represents the maximum deviation of the control path, the sign of y would be transmitted to the steering control system on the tractor.

If $|AB/ct_t - h| < y$, then the previous control signal would be changed to zero or left at zero if it was zero.

The routine that follows immediately, is the one that

will yield the cotangent of angles between 0 and $\pi/4$. A minor alteration extends the range of angles from 0 to $\pi/4$ to 0 to $\pi/2$. The extended program is referred to later. This routine was prepared in order to establish the time required to calculate the cotangents to the required precision using the above algorithm.

			INPUT=13
			OUTPUT=14
		*7	
0007	5600	5600	
		*1000	
			TLS
1000	4407	START,	JMS I 7
1001	0013		INPUT
1002	6240		FPUT A1
1003	0013		INPUT
1004	6243		FPUT A2
1005	0013		INPUT
1006	6246		FPUT B2
1007	0013		INPUT
1010	6251		FPUT B3
1011	0013		INPUT
1012	6265		FPUT C1
1013	0000		FEXT
1014	4407	HERE,	JMS I 7
1015	0013		INPUT
1016	6254		FPUT R0
1017	5254		FGET R0
1020	3254		FMPY R0
1021	6257		FPUT R02
1022	5251		FGET B3
1023	4257		FDIV R02
1024	2246		FSUB B2
1025	1257		FADD R02
1026	6262		FPUT TEMP
1027	5240		FGET A1
1030	3257		FMPY R02
1031	2243		FSUB A2
1032	4262		FDIV TEMP
1033	1265		FADD C1
1034	4254		FDIV R0
1035	0014		OUTPUT
1036	0000		FEXT
1037	5214		JMP HERE

1040	0000	A1,	0
1041	0000		0
1042	0000		0
1043	0000	A2,	0
1044	0000		0
1045	0000		0
1046	0000	B2,	0
1047	0000		0
1050	0000		0
1051	0000	B3,	0
1052	0000		0
1053	0000		0
1054	0000	R0,	0
1055	0000		0
1056	0000		0
1057	0000	R02,	0
1060	0000		0
1061	0000		0
1062	0000	TEMP,	0
1063	0000		0
1064	0000		0
1065	0000	C1,	0
1066	0000		0
1067	0000		0

A1	1040
A2	1043
B2	1046
B3	1051
C1	1065
HERE	1014
INPUT	0013
OUTPUT	0014
R0	1054
R02	1057
START	1000
TEMP	1062

The time required for the computer to produce the cotangent of the angular input was .16 milliseconds. Since two cotangents are required as well as some operations in logic, the total time required will be more than twice the time determined above.

The next routine is designed to incorporate all the logic required in the operation of the system with the exception of filling in the time delay that would nearest

ROUTINE #2

	INPUT = 13	
	OUTPUT = 14	
*7		
5600		
*1000		
START,	TLS	
	JMS I 7	
	INPUT	
	FPUT A1	
	INPUT	
	FPUT A2	
	INPUT	
	FPUT B2	
	INPUT	
	FPUT B3	
	INPUT	
	FPUT C1	
INPUT	INPUT	
	FPUT SC	scale factor
	INPUT	
	FPUT P02	pi over 2
	INPUT	
	FPUT P04	pi over 4
HERE,	CLA	clear accumulator
	IST	strobe input buffer
	CIA	2's complement of contents
	JMS I 7	Call floating point pack.
	INPUT	
	FPUT R	store R
	FGET R	
	FMPY SC	mult by scale factor
	FSUB P04	$FAC = R - \pi/4$
	FEXT	
	CLA	
	TAD HORD	add high order mantissa
	SPA	skip with positive ac
	JMP LOW	$R_0 = R$
	SMA	
	JMP COT	$\cot R = 1$
	JMS I 7	
	FGET P02	
	FSUB R	
	FPUT R0	$R_0 = \pi/2 - R$
	FEXT	
	CIA	
	DCA	
	JMP COMP -1	
	JMS I 7	
LOW,	JMS I 7	
	FPUT R0	$R_0 = R$
COMP,	FGET R0	
	FMPY R0	Square R_0

	FPUT R02	
	FGET B3	
	FDIV R02	
	FSUB B2	
	FADD R02	
	FPUT TEMP1	store denominator
	FGET A1	
	FMPY R02	compute numerator
	FSUB A2	
	FDIV TEMP1	
	FADD C1	compute and store F
	FPUT TEMP2	
	FEXT	
	TAD CHECK	
	SZA	check whether R was less
	JMP OUT	than $\pi/4$
	JMS I 7	no
	FGET R0	$\cot R = R_0/F$
	FDIV TEMP2	
	OUTPUT	
	FEXT	
	CLA	
	DCA CHECK	
	JMP HERE	
OUT,	JMS I 7	yes
	FGET TEMP2	
	FDIV R0	$\cot R = F/R_0$
	OUTPUT	
	FEXT	
	JMP HERE	
COT,	CIA	
	JMS I 7	
	INPUT	
	FPUT TEMP3	$\cot R = 1$
	FGET TEMP3	
	OUTPUT	
	FEXT	
	CLA	
	DCA CHECK	
	JMP HERE	

make the delay time an integral multiple of the sampling period of the detector system.

Explanations to the right of the instructions are to clarify the purpose of the instructions. Making the estimate of the time required to complete the operations designated, on the basis of 1.5 microseconds for logic operations and 21.0 and 36.5 microseconds for multiplication and division respectively, the time required is less than 0.25 seconds. Thus if a sampling frequency of 4 samples per second is adequate, the computer is well able to meet the specification.

CHAPTER IV

GENERAL ANALYSIS OF THE CONTROL SYSTEM

For analysis purposes, using approximations to the transfer functions of the equipment is the rule rather than the exception. In the last chapter, approximations to the transfer functions of the equipment were established. Now the procedure is to reestablish some order to the control system so that the analysis can be carried out. The block diagram of the control system, Fig. 4.1, is the first step in that direction.

The sampler is a switch that is activated by a pulse that originates in the detector amplifier circuitry. The time lag e^{bST} , is the lag imposed not only by the computer but also the transmitter; the transmission of infrared rays; the delay of few micro-second duration for which the data await transfer to the accumulator, in the input buffer register; and the time lag of the response of the guidance system on the tractor. Although lumping the time lags into one is not always a particularly good approximation, it will serve in this case where the undamped natural frequency of the system is relatively low. Further, the simplification achieved in the analysis, warrants the loss in accuracy in the approximation of the response of the system to a disturbance.

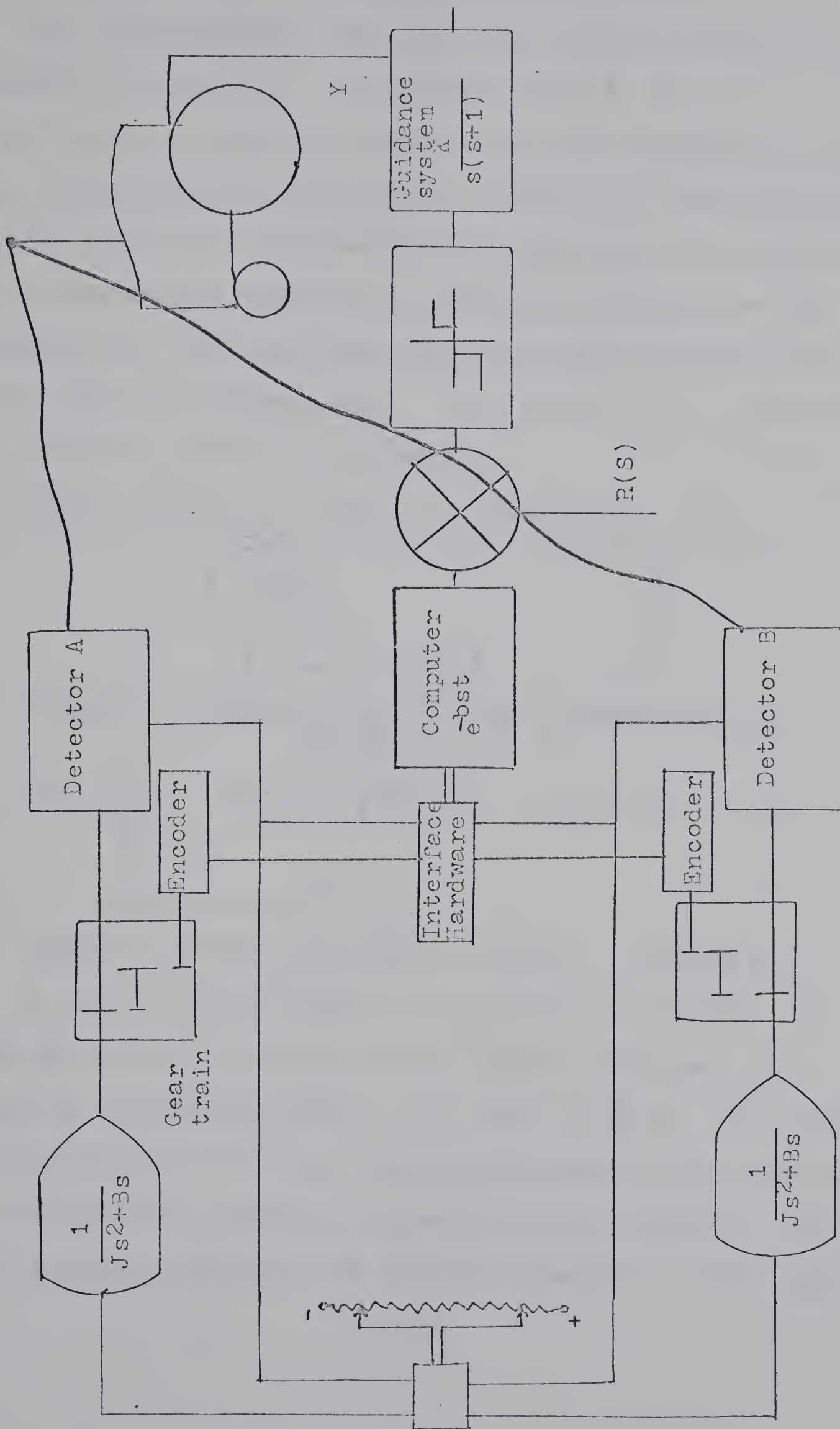


Figure 4.1 BLOCK DIAGRAM OF COMPLETE CONTROL SYSTEM

4.2 GAIN LIMIT OF A SECOND-ORDER CONTROL SYSTEM

The block diagram, Fig 4.2, is a representation of the transfer functions of the elements seen in Fig. 4.1. Fig. 4.3 is a simplification of the block diagram in Fig. 4.2. The essential difference is the transfer of the transportation delay to a point after the comparator and replacing the synchronized samplers by a single sampler after the comparator. This does not alter the sense of the system since $R(S)$ is constant over a time period T_p T , where T is the time period of the sampler.

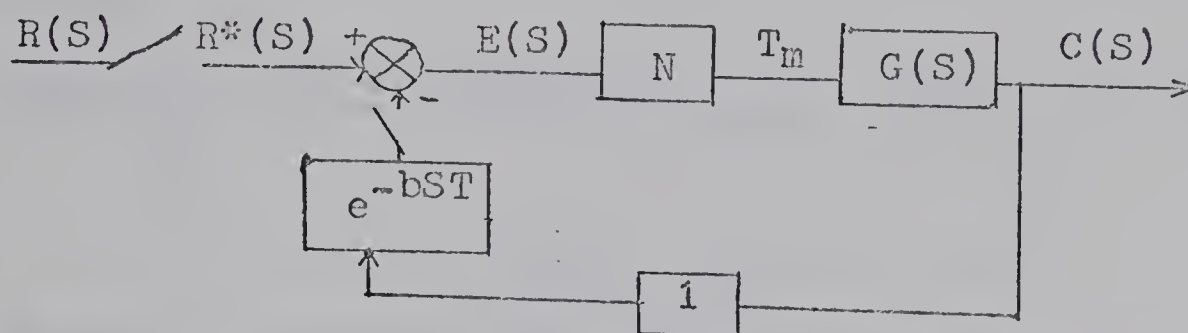


FIGURE 4.2 BLOCK DIAGRAM OF THE ELEMENTS PRESENT IN FIGURE 4.1

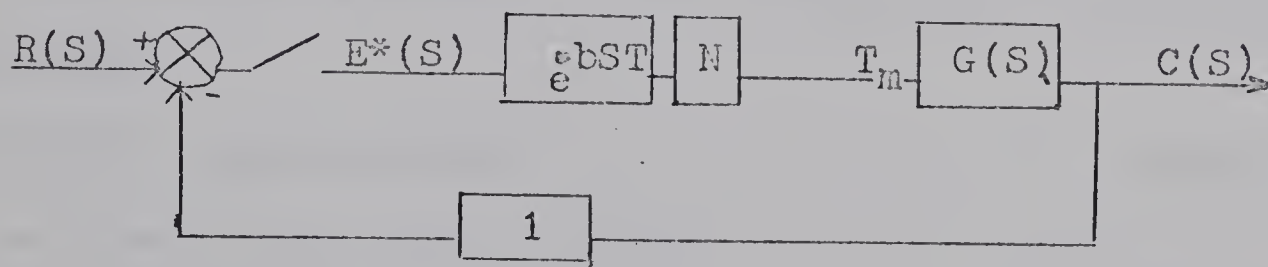


FIGURE 4.3 SIMPLIFIED BLOCK DIAGRAM OF FIGURE 4.2

T_m in the above figures, is a nonlinear coefficient that describes the output of the relay with dead zone, which is represented in Fig. 4.2 and 4.3 by N . The linear forward function $G(S)$ is the approximation of the transfer function of the guidance mechanism on the tractor. For this general analysis, the transfer function of the steering

mechanism is $T_m v^2 / s(s+1)$. The unity damping is provided by the gyrocompass controlling a potentiometer in a servo circuit.

It is noted that the transfer function of the detector drive system does not appear in the transfer function of the above system. It is assumed, for analysis purposes, that the two samplers are synchronized however.

4.3 GAIN LIMIT OF A SAMPLED DATA CONTROL SYSTEM WITH ZERO-ORDER HOLD

Consider the second-order sampled data control system in Fig. 4.5.

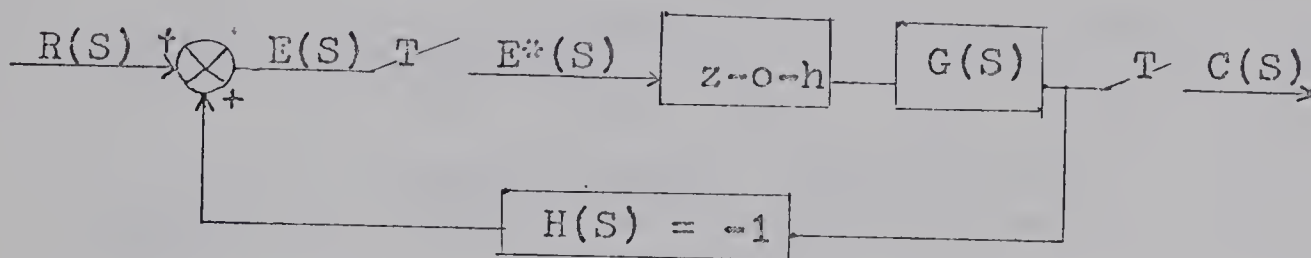


FIGURE 4.5 SAMPLED DATA SYSTEM WITH ZERO-ORDER HOLD

The transfer function of the forward path is $G_{ho}G(S)$.

That means

$$C^*(S) = G_{ho}G^*(S)E^*(S) \quad 4.3.1$$

The z-transform of $C(S)$ is then

$$C(Z) = \frac{G_{ho}G(Z)}{1 + HG_{ho}G(Z)} R(Z) \quad 4.3.2$$

This can be written as

$$\frac{C(Z)}{R(Z)} = \frac{G_{ho}G(Z)}{1 + HG_{ho}G(Z)} \quad 4.3.3$$

which is recognized to be the z-transform of the system in Fig. 4.3. Adding the z-o-h, G_{ho} , to the circuit with a sampler gives an additional term $\frac{1-e^{-ST}}{S}$ to the forward transfer function.

Now

$$G_{ho}G(s) = \frac{1 - e^{-st}}{s^2(s + 1)} \quad 4.3.5$$

Since

$$\frac{C(Z)}{R(Z)} = \frac{G_{ho}G(Z)}{1 + HG_{ho}G(Z)} \quad 4.3.6$$

$$\frac{C(Z)}{R(Z)} = \frac{(1 - z^{-1})}{1} \cdot \frac{G_{ho}G(s)}{1 + HG_{ho}G(s)} \quad 4.3.7$$

in which

$$ZG_{ho}G(s) = ZHG_{ho}G(s) = \mathcal{Z} \left[\frac{K}{s^2(s + 1)} \cdot \frac{(1 - z^{-1})}{1} \right] \quad 4.3.8$$

$$G_{ho}G(Z) = \frac{(1 - z^{-1})K}{1} \left[\frac{Tz}{(z - 1)^2} - \frac{(1 - e^{-T})z}{(z - 1)(z - e^{-T})} \right] \quad 4.3.9$$

$$G_{ho}G(Z) = \frac{K((T-1+e^{-T})z - Te^{-T} + 1 - e^{-T})}{(z-1)(z-e^{-T})} \quad 4.3.10$$

The characteristic equation obtained by setting the denominator of Eq. 4.3.6 equal to zero is

$$(z-1)(z-e^{-T}) + K((T-1+e^{-T})z - Te^{-T} + 1 - e^{-T}) = 0 \quad 4.3.11$$

To obtain the relationship between K and T, they are left unspecified. It is also necessary to change the equation to the r-transform in order to apply the Routhian technique. This is done by substituting $(r+1)/(r-1)$ for z in Eq. 4.3.11.

$$\frac{2}{(r-1)} \frac{(r+1)}{(r-1)} e^{-T} + \frac{K(T-1+e^{-T})(r+1)}{(r-1)} - Te^{-T} + 1 - e^{-T} = 0 \quad 4.3.12$$

Multiplying Eq. 4.3.12 by $(r-1)^2$ yields

$$2r+2-2re^{-T}+2e^{-T}+K(T-1+e^{-T})r - K(T-1+e^{-T}) - (Te^{-T}-1+e^{-T})r + 2K(Te^{-T}-1+e^{-T})r - K(Te^{-T}-1+e^{-T}) = 0 \quad 4.3.13$$

The Routhian array of Eq. 4.3.13 is

$$\begin{array}{l} r^2 \quad KT(1-e^{-T}) \qquad \qquad \qquad 2(1+e^{-T})-K(Te^{-T}-2+2e^{-T}+T) \\ r^1 \quad 2-2e^{-T}+2K(Te^{-T}-1+e^{-T}) \\ r^0 \quad 2(1+e^{-T})-K(Te^{-T}-2+2e^{-T}+T) \end{array}$$

Since there are no negative signs in the first column of the array, it is concluded that there are no negative roots for a certain range of values of K and T. The Table X shows numerically the relationship between specified values of T and K in the r^1 and r^0 coefficients of the array. The data is displayed graphically in Fig. 4.6.

TABLE X MAXIMUM GAIN VALUE FOR SPECIFIED T

T	1	.9	.8	.7	.6	.5	.4	.3	.2
K_{r1}	2.39	2.67	2.88	3.25	3.70	4.37	5.88	7.40	11.3
K_{r2}	27	35	52	75					

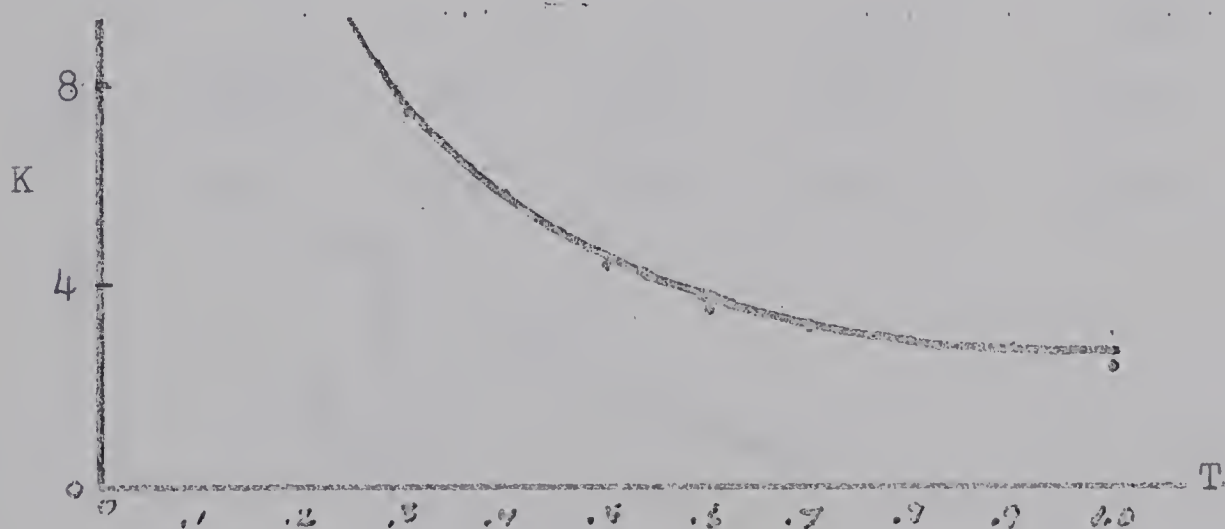


FIGURE 4.6 GAIN VERSUS SAMPLING PERIOD

Now suppose the hold function $\frac{(1-e^{-Ts})}{s}$, is delayed by $e^{0.5T}$ for example. What is the net effect on the gain?

$$G_{ho}G(s) = \frac{(1-e^{-sT})e^{0.5sT}}{s} \left(\frac{K}{s(s+1)} \right) \quad 4.3.14$$

$$G_{ho}G(Z) = \frac{(1-z^{-1})\bar{e}^{0.5T}}{s} \frac{K}{s^2(s+1)} \quad 4.3.15$$

The characteristic equation $1+HG_{ho}G(Z) = 0$, becomes

$$(z-1)(z-e^{-T}) + K\bar{e}^{0.5T}((T-1+e^{-T})z + Te^{-T}+1-e^{-T}) = 0 \quad 4.3.16$$

Conversion to the r-transform results in the equation

$$\begin{aligned} \bar{e}^{0.5T}KT(1-e^{-T})r + r(2(1+e^{-T})+\bar{e}^{0.5T}K(Te^{-T}-2+2e^{-T}+T)) \\ + 2(1+e^{-T})-\bar{e}^{0.5T}K(Te^{-T}-2+2e^{-T}+T)=0 \end{aligned} \quad 4.3.17$$

The effect of the transportation delay, $e^{-0.5T}$, in the system is to multiply K by a factor of 1/1.6487 (without sacrificing stability) when T is one second. Table XI shows numerically, the relationship between T and the amplification factor when the transportation delay is $\frac{1}{2}$ sec. The data is graphically displayed in Fig. 4.5.

TABLE XI MAXIMUM K FOR SYSTEM WITH TRANSPORTATION LAG

T	1	.75	.50	.25
$F(e^{\frac{1}{4}ST})$	1.28	1.35	1.65	2.71
K_{max}	1.87	2.22	2.65	3.40

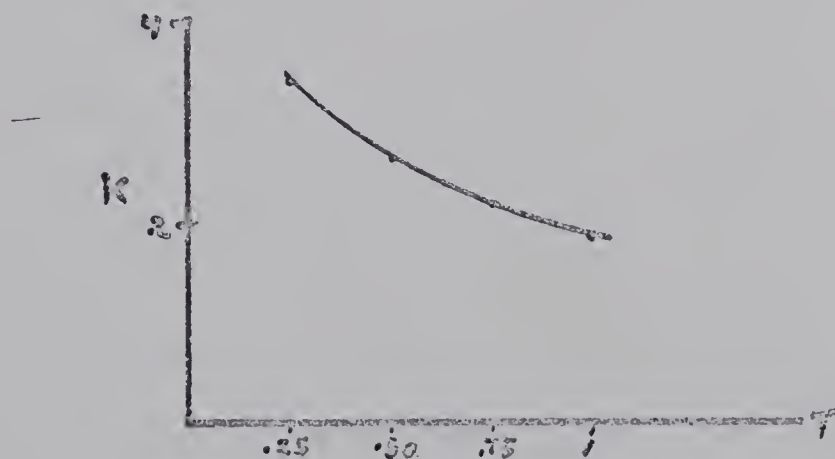


FIGURE 4.5 GAIN VERSUS SAMPLING PERIOD IN A SYSTEM WITH TRANSPORTATION LAG.

From the examination of the system with transportation lag, it is found that the system is stable for certain limits of gain. Since the gain is $T_m v^2/s(s+1)$, in which T_m is the steering constant and v is the velocity of the trac-

tor, it is seen that the limits of gain can be realized by decreasing either the steering constant or the velocity of the tractor. Hence, the system theoretically works satisfactorily within limits of speed and turning radius that are practical in field operations.

CHAPTER V

RESPONSE DETERMINATION

It was established that the control system may be designed to be stable for some operating conditions. The operating conditions that the system would be subjected to in practice, however, may be nowhere near the stable range. Operating in the realm of possibility that the coefficients that are derived theoretically, are also correct for a practical system, observations of the response of the control system that has only the initial conditions as disturbances, are of interest. It is recognized that in practice there is virtually no linear system; however, approximations of practical operating conditions by linear functions, will lead to reasonable conclusions regarding the performance of the system provided the limits of operation do not deviate from the region of approximation.

With the above realization, the operating conditions listed below are derived.

Operating Conditions:

- (1) $\frac{1}{4}$ mile square plot, Fig. 5.1
- (2) operating at an extreme point
- (3) it is desirable to leave a headland of 20 ft.
- (4) there is unobstructed view to all parts of the plot
- (5) the tractor is located at point P, 20 ft. in from an edge of the plot

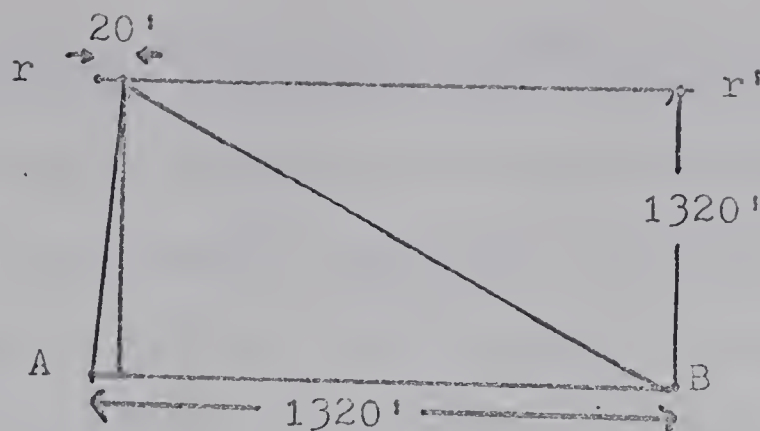


FIGURE 5.1 OPERATING POSITION

- (6) there are observation points at A and B, the end points of the reference line, 1320' apart and 1320' from the desired path rr'
- (7) the operating limits are $1319' < h < 1321'$
- (8) the initial conditions are $x_1 = 1321$ and $x_2 = 0$
- (9) the modulated sampling period T , is 1 second
- (10) the turning radius of the tractor is 20 ft.
- (11) the velocity of the tractor is 8 ft./sec
- (12) the gyrocompass feedback is -1
- (13) from (10) and (11) the steering constant T_m is found to be .05 radians/foot.
- (14) the lumped time lag is .25 sec.

The transfer function of the guidance system on the tractor is $G_1(S) = 64/s(s+1)$. The switching limits of the relay with dead zone are ± 1.0 . $R(t_0)$ is a unit step. The control system is represented in the block diagram of Fig. 5.2

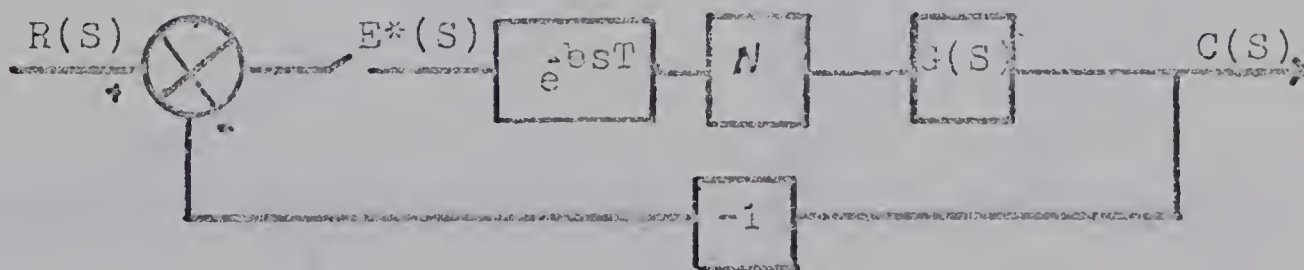


FIGURE 5.2 BLOCK DIAGRAM OF THE CONTROL SYSTEM

5.1.2 ANALYSIS OF THE SAMPLED-DATA SYSTEM WITH NONLINEARITY BY THE STATE TRANSITION TECHNIQUE WITH VARIABLE GAIN CONCEPT

The variable gain concept with the state transition method, permits a systematic synthesis procedure for discrete-data control systems. The variable gain concept is a method of representing a nonlinearity by a variable gain amplifier with gain $K(kT) = K_k$ during the sampling period $kT \leq t \leq (k+1)T$. Use is made of the transition flow graph and equations, thus the method is essentially numeric.

$$K_k \text{ is defined as } K_k = \frac{m(kT+)}{h(kT+)} \quad 5.1.1$$

where m is the output of the nonlinear element and h is the input to the nonlinear element from the z-o-h. An element with similar characteristics to the one used by MacHardy¹¹ in the continuous data system will be considered.

The nonlinearity is a relay type function with dead zone and with its input-output characteristics described by

$$m(t) = \begin{cases} +.05 & \text{if } h(t) \geq 1 \\ -.05 & \text{if } h(t) \leq -1 \\ 0.00 & \text{if } -1.0 < h(t) < 0.1 \end{cases}$$

From the transition flow graph, Fig. 5.3, of the nonlinear sampled-data control system in Fig. 5.2, the transition equations are obtained in the following vector

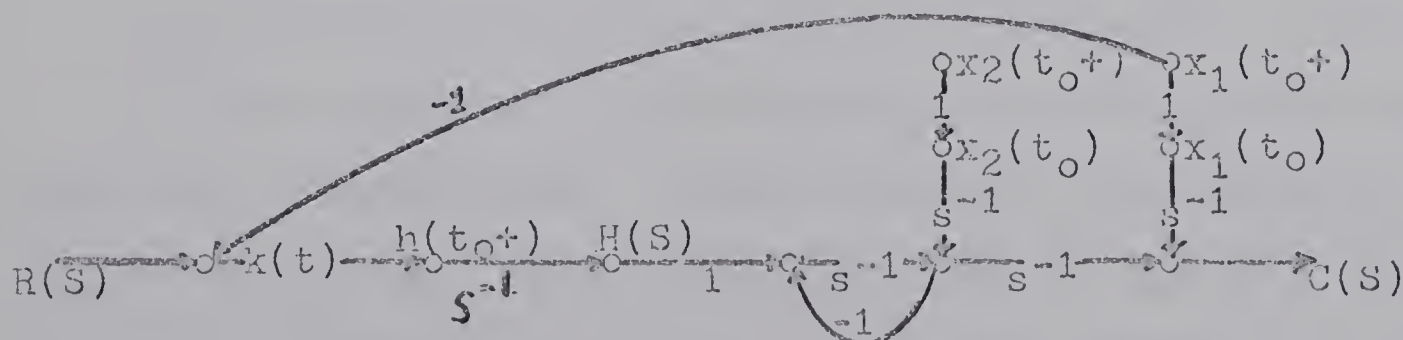


FIGURE 5.3 SIGNAL FLOW GRAPH OF THE SAMPLED-DATA CONTROL SYSTEM

matrix form:

$$x(t) = M((t-t_0), K(t_0))x_1(t_0) + N((t-t_0), K(t_0))r(t_0) \quad 5.1.3$$

When $t=(k+1)T$ and $t_0=kT$, Eq. 5.1.3 becomes

$$x((k+1)T) = M(T, K_k)x_1(kT) + N(T, K_k)r(kT) \quad 5.1.4$$

The values of K_k are given by Eq. 5.1.1 in which

$h(kT+)$ is determined from

$$h(kT+) = r(kT) - x_1(kT) \quad 5.1.5$$

and $m(kT+)$ is determined from the nonlinear characteris-

tics once $h(kT+)$ is known. After the values of K_k

during each sampling period are determined, Eq. 5.1.4 gives

the solution of the state variables at the sampling

instants.

For the state transition flow graph of the system

drawn in Fig. 5.3, the transition equations are

$$x_1((k+1)T) = (1 - K_k \cdot v^2(T-1+e^{-T}))x_1(kT) + v(1-e^{-T})x_2(kT) \\ + K_k v^2(T-1+e^{-T})r(kT) \quad 5.1.6$$

$$x_2((k+1)T) = -K_k v(1-e^{-T})x_1(kT) + e^{-T}x_2(kT) \\ + K_k v(1-e^{-T})r(kT) \quad 5.1.7$$

If the initial conditions, $T=1$ sec., $v=8$ ft/sec, and $r(kT)=0$

are substituted in Eq. 5.1.6 and 5.1.7, they become

$$x_1((k+1)T) = (1 - 64 \cdot 0.368K_k)x_1(kT) + 8 \cdot 0.632x_2(kT) \quad 5.1.8$$

$$x_2((k+1)T) = (-8 \cdot 0.632K_k)x_1(kT) + 0.368x_2(kT) \quad 5.1.9$$

In the analysis of the control system the time delay must also be considered. Recall that the time delay is the period between the time the position of the tractor is sampled and the time the signal reaches the guidance system

on the tractor. It was determined that the lag can be limited to approximately 0.25 seconds in Chapter IV. To cope with the delay of 0.25 seconds in the analysis, the K_k was calculated on the basis of the conditions of X_1 at the instant when $t=3/4(T)$. The new K_k was then substituted in the equations 5.1.8 and 5.1.9, and the new X_1 and X_2 were calculated on the basis of the values of X_1 and X_2 determined at the end of the last sampling period T .

The set of equations, Eq. 5.1.8 and 5.1.9, were programmed on the APL/360 and solutions for the first 20 intervals or sampling periods were calculated. The routines used on the APL/360 computer and the results produced are found in the Appendices. Fig. 5.4, 5.5, and 5.6 are plots of the displacement as a function of time.

5.2 DISCUSSION

Since there is no reason to anticipate occurrence of frequencies less than the sampling frequency, the values of X_1 can be connected by smooth curves as was done in Figures 5.4, 5.5, and 5.6. The smooth curves produced, are a reasonable estimation of the performance of the system between the sampled instants. The equations were not solved for more than 20 sampling periods because information that could be obtained by carrying the solutions out to infinity would be questionable on the basis of the approximations that were made to obtain the mathematical description of the system.

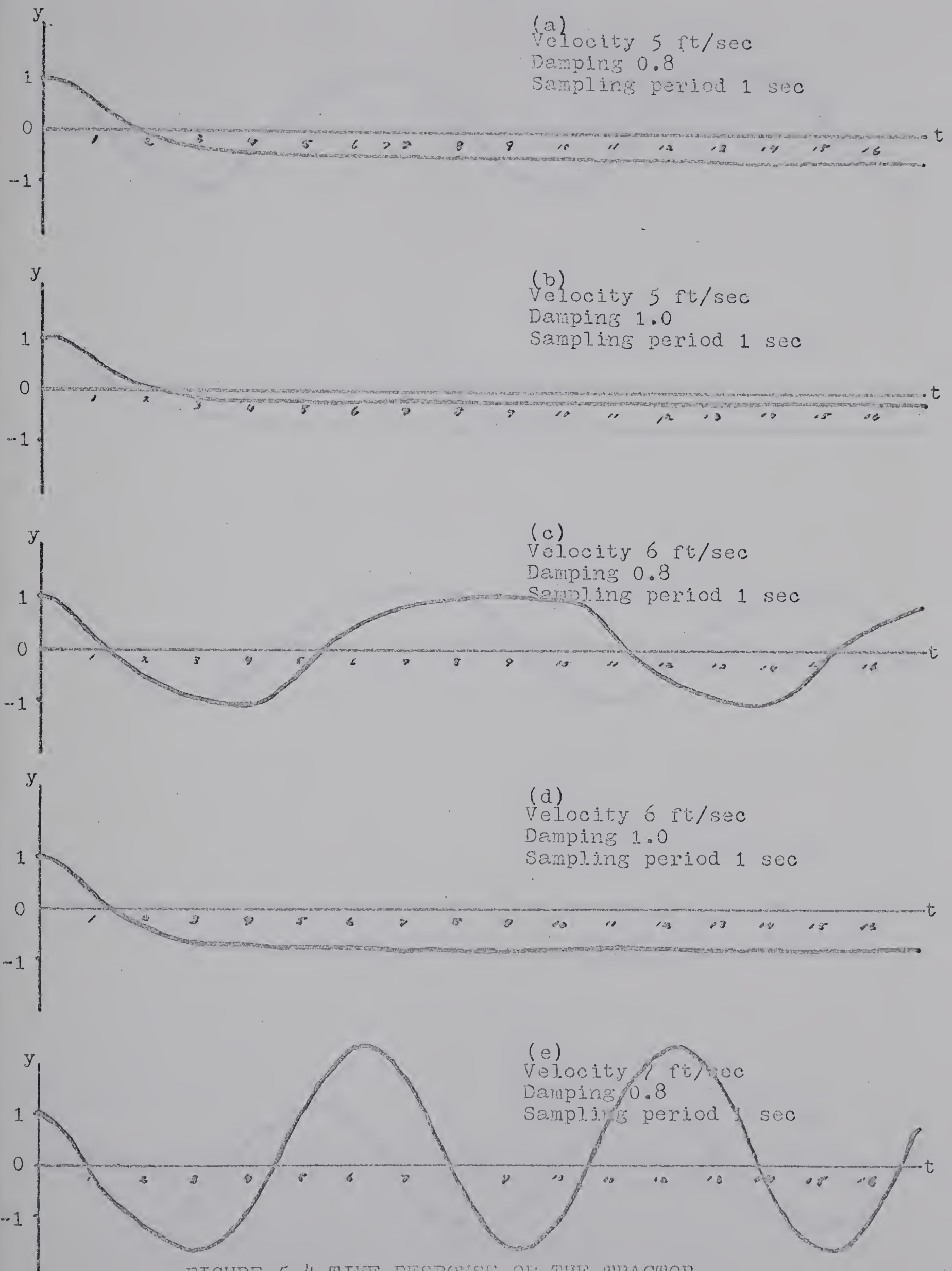


FIGURE 5.4 TIME RESPONSE OF THE TRACTOR

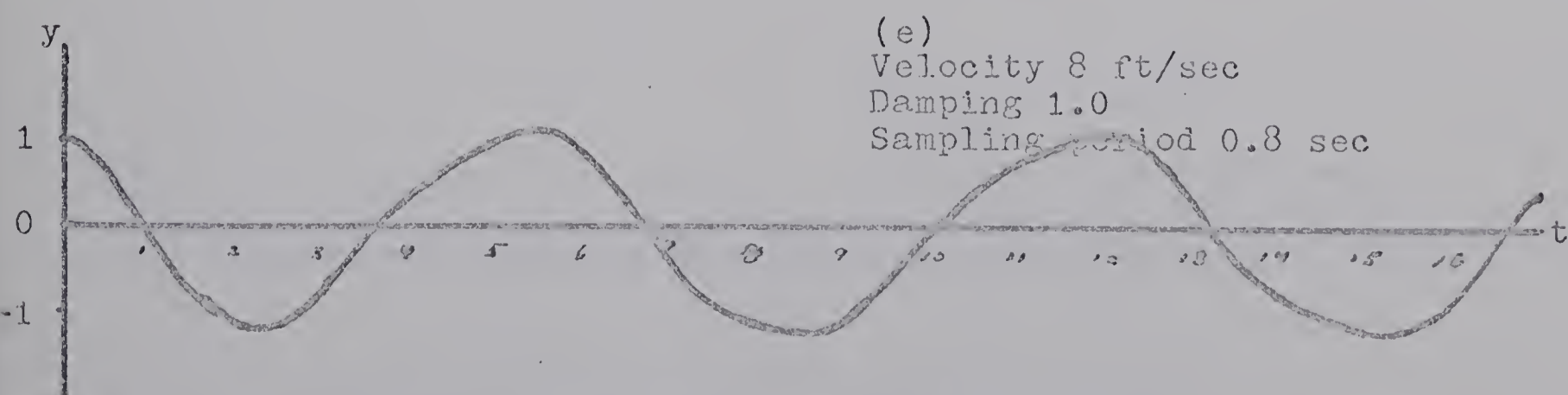
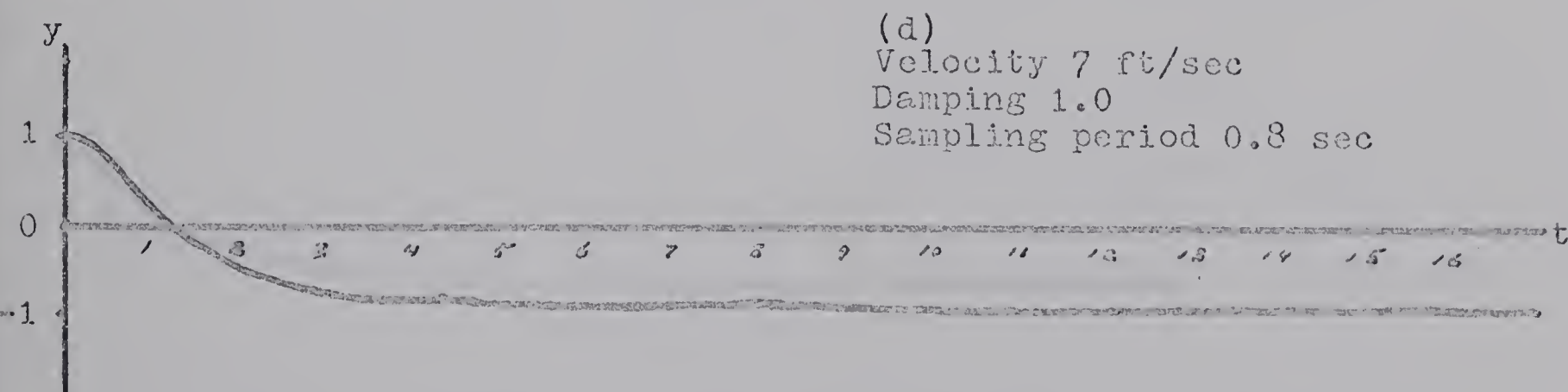
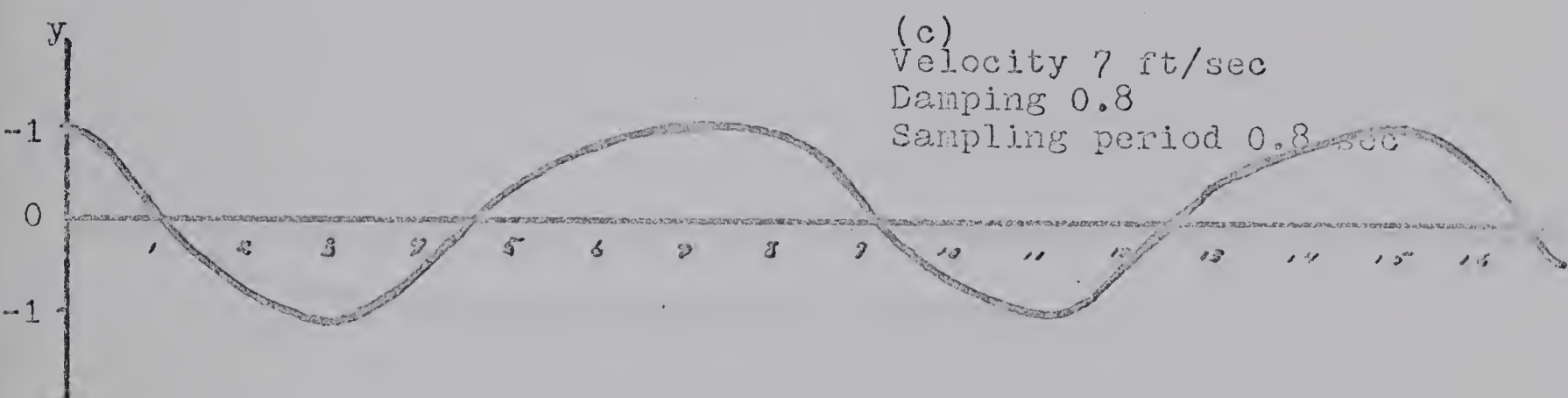
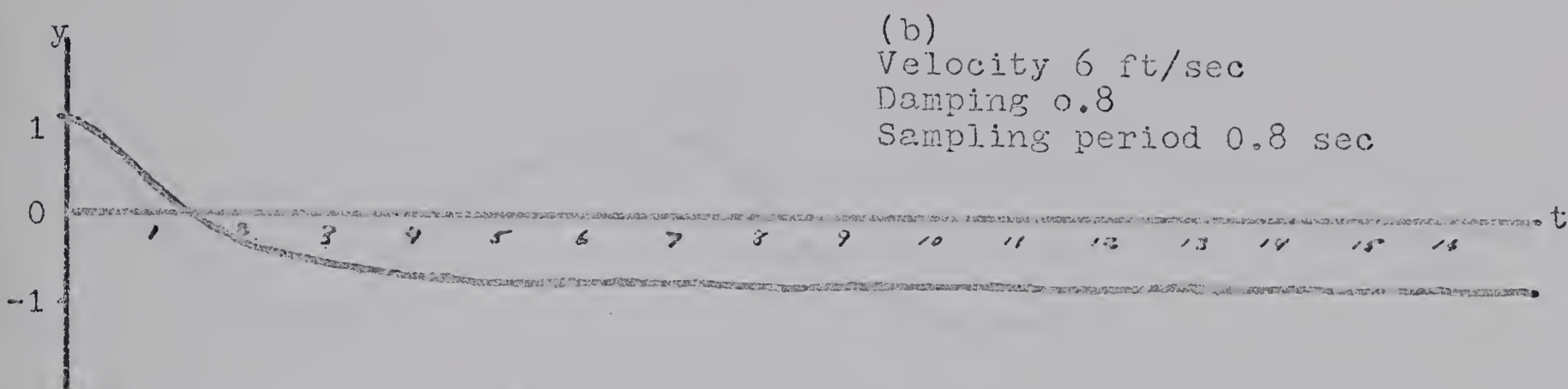
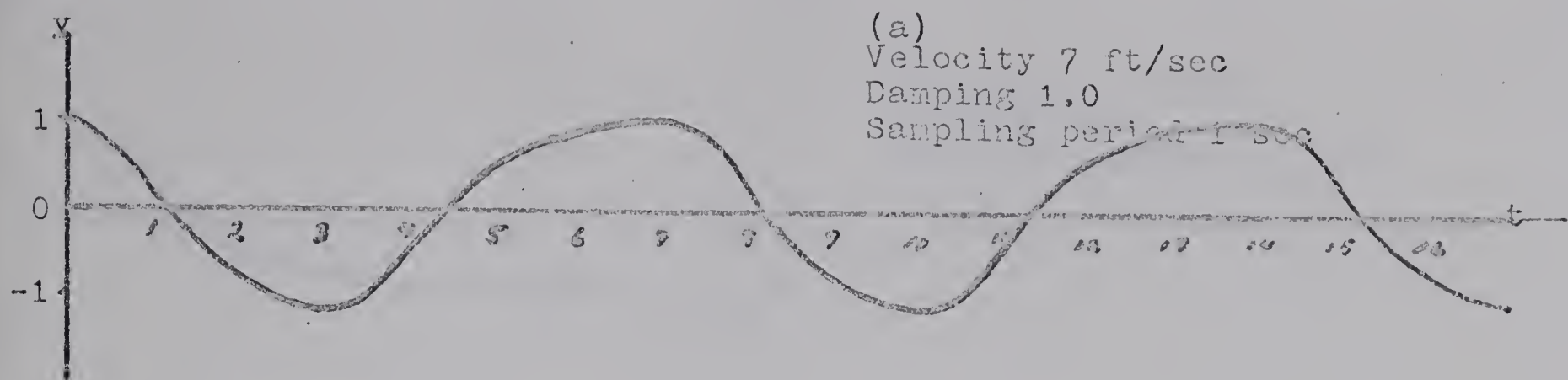


FIGURE 5.5 TIME RESPONSE OF THE TRACTOR

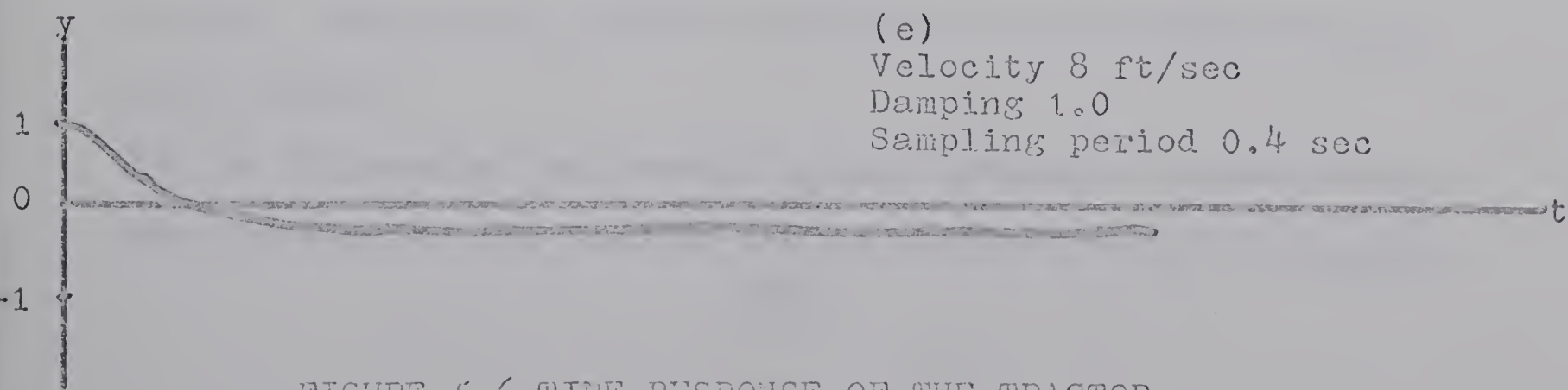
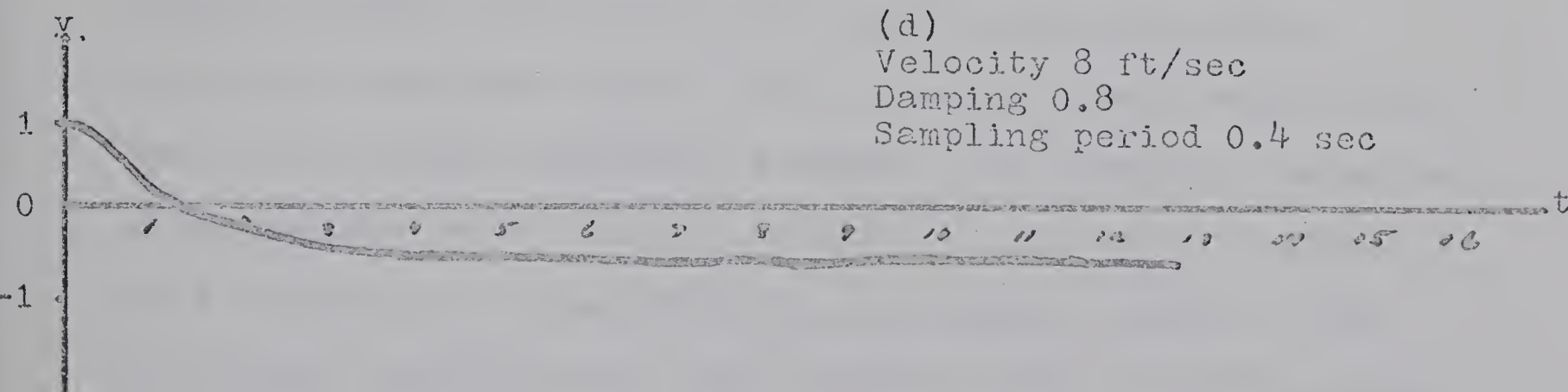
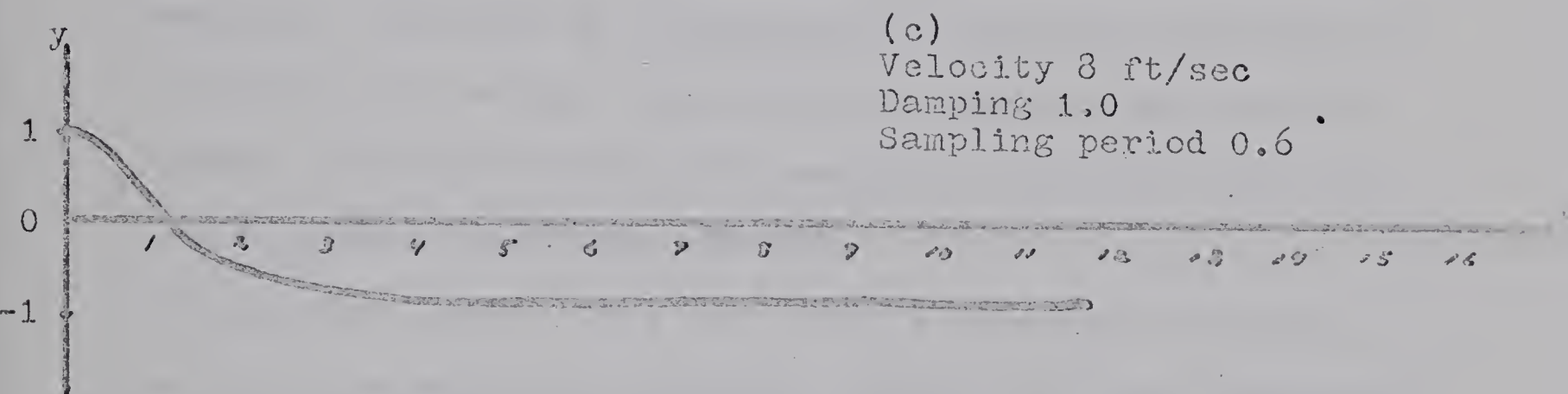
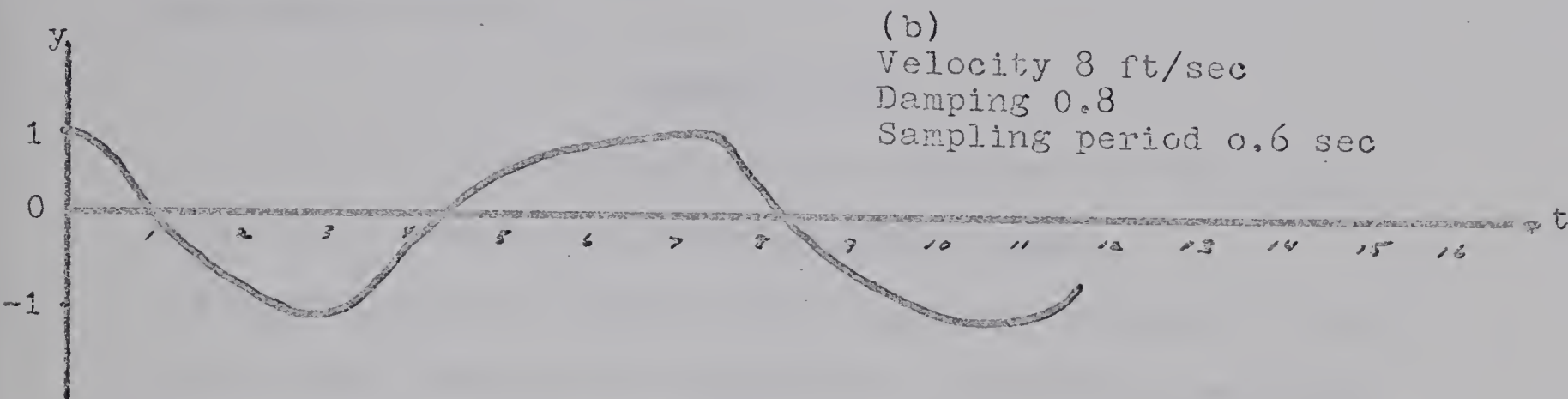
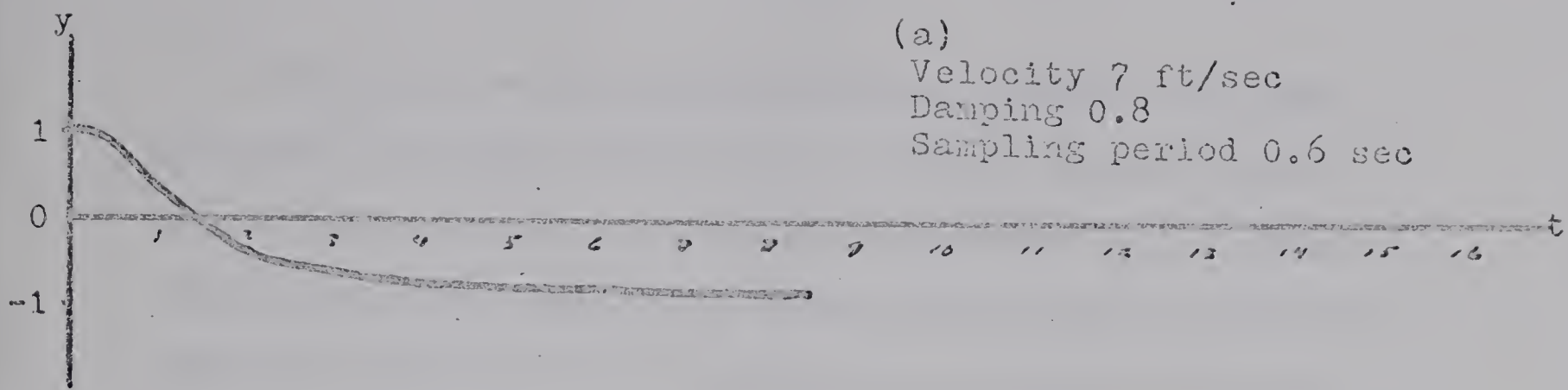


FIGURE 5.6 TIME RESPONSE OF THE TRACTOR
(63)

Solutions to the equations were obtained for only one set of initial conditions; a lateral displacement, X_1 , of one foot and zero lateral velocity, X_2 . Initial conditions other than those above would lead to different transient solutions or to different periodic solutions, according to Kuo¹⁴.

CONCLUSIONS

Based on observations of the solutions to the set of Eq's. 5.1.8 and 5.1.9, it is concluded that:

- (1) Operating at 8 ft/sec and a sampling frequency of once per second, the control system has an unstable transient response. However by increasing the sampling frequency to 2.5 times per second, the transient solution can be made stable. Thus increasing the sampling frequency leads to a more stable transient response.
- (2) Besides leading to a more stable transient solution, an increased sampling frequency results in lower overshoot, smaller steady state error, and shorter settling time.
- (3) Wider dead zone limits lead to larger steady state error but a more stable transient response. For example, the system when operated with a dead zone limit of ± 2 feet, is stable at a velocity of 8 feet/second, and sampling period 1 sec.
- (4) Longer turning radii lead to more stable response, lower natural frequencies, lower overshoots, and to larger steady state errors.
- (5) By increasing the demands on the detection system so that the position of the tractor can be sampled every 0.4 seconds,

the tractor has a stable transient response. However, to increase the sampling frequency, it is necessary to reduce the field scanned by the detector in order to maintain the same relative position of the tractor exhaust muffler in the detector field when the 50% level of response is reached. For example: a field width of $12\frac{1}{2}^{\circ}$ can be scanned when the sampling frequency is once per second, the region that can be scanned is reduced in width to 5° (due to the maximum velocity of scan of .22 radians per second).

ADDENDUM

The control of a tractor by the method described in this thesis is possible; how practical, however, remains to be judged in each application under consideration. Field operations with wide implements that require control in overlap of less than one foot, can be carried out by the automatically controlled tractor, but between the rows cultivation (for example) requires greater precision than that designed into the system described in this thesis.

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      'APPENDIX' ;1;      'DATA ZONE ONE FOOT'
      VGLL[5]V
7 CRI
[1]  T←0.6
[2]  STA:V←5
[3]  STP:A←0.8
[4]  STAT:K←0
[5]  ('DAMPING' ;A;' VELOCITY' ;V;' SAMPLING PERIOD' ;T)
[6]  'NO X11 X21 SX1 SX2 N'
[7]  T←T-0.25
[8]  D←0
[9]  X1←1
[10] X2←0
[11] C←1
[12] K←0
[13] P←0
[14] SX1←0
[15] SX2←0
[16] P←(T-(1-**-A×T):A):A
[17] C←(1-**-A×T):A
[18] STAR:X11←(X1-X1×K×P×V×2)+(V×C×X2)+K×P×D×V×2
[19] X21←(-K×V×C×X1)+(X2×**-A×T)+K×C×V×D
[20] C←(1-**-A×P):A
[21] P←(P-C):A
[22] SX1←(X1-X1×K×P×V×2)+(V×C×X2)+K×P×D×V×2
[23] SX2←(-K×V×C×X1)+(X2×**-A×T)+K×V×C×D
[24] C,2 PPE X11,X21,SX1,SX2,P,K
[25] P←-SX1
[26] I←((~IV), (I←E≥2), I←F≤-1)/ 0 0.05 -0.05
[27] K←N:I
[28] X1←X11
[29] X2←X21
[30] C←C+1
[31] →(C≤30)/STAR
[32] ST:A←A+0.2
[33] →(A≤1)/STAT
[34] V←V+1
[35] →(V≤6)/STR
[36] T←T-0.2
[37] →(T≤0.25)/LD
[38] →STA
[39] PD:'END'

```


GRL

DAMPING 0.8 VELOCITY SAMPLING PERIOD 1

TO X11 X21 SX1 SX2 Y

1	1	0	1	0	0	0
2	0.51	-0.17	0.71	-0.14	-0.05	0.05
3	-0.03	-0.03	0.03	-0.03	0	0
4	-0.35	-0.03	-0.3	-0.04	0	0
5	-0.46	-0.02	-0.44	-0.02	0	0
6	-0.52	-0.01	-0.51	-0.01	0	0
7	-0.54	0	-0.54	0	0	0
8	-0.55	0	-0.55	0	0	0
9	-0.56	0	-0.56	0	0	0
10	-0.56	0	-0.56	0	0	0
11	-0.56	0	-0.56	0	0	0
12	-0.56	0	-0.56	0	0	0
13	-0.56	0	-0.56	0	0	0
14	-0.56	0	-0.56	0	0	0
15	-0.56	0	-0.56	0	0	0
16	-0.56	0	-0.56	0	0	0
17	-0.56	0	-0.56	0	0	0
18	-0.56	0	-0.56	0	0	0
19	-0.56	0	-0.56	0	0	0
20	-0.56	0	-0.56	0	0	0

DAMPING 1 VELOCITY SAMPLING PERIOD 1

TO X11 X21 SX1 SX2 Y

1	1	0	1	0	0	0
2	0.54	-0.16	0.72	-0.13	-0.05	0.05
3	0.04	-0.06	0.12	-0.07	0	0
4	-0.14	-0.02	-0.11	-0.03	0	0
5	-0.21	-0.01	-0.2	-0.01	0	0
6	-0.24	0	-0.23	0	0	0
7	-0.24	0	-0.24	0	0	0
8	-0.25	0	-0.25	0	0	0
9	-0.25	0	-0.25	0	0	0
10	-0.25	0	-0.25	0	0	0
11	-0.25	0	-0.25	0	0	0
12	-0.25	0	-0.25	0	0	0
13	-0.25	0	-0.25	0	0	0
14	-0.25	0	-0.25	0	0	0
15	-0.25	0	-0.25	0	0	0
16	-0.25	0	-0.25	0	0	0
17	-0.25	0	-0.25	0	0	0
18	-0.25	0	-0.25	0	0	0
19	-0.25	0	-0.25	0	0	0
20	-0.25	0	-0.25	0	0	0

DAMPING0.8VELOCITY6SAMPLING PERIOD1

NO	X11	X21	SX1	SX2	K
1	1	0	1	0	0
2	0.3	-0.21	0.58	-0.17	-0.05 0.05
3	-0.55	-0.09	-0.4	-0.11	0 0
4	-0.94	-0.04	-0.87	-0.05	0 0
5	-1.11	-0.02	-1.08	-0.02	0 0
6	-0.47	0.2	-0.74	0.13	0.05 0.05
7	0.38	0.09	0.23	0.11	0 0
8	0.76	0.04	0.69	0.05	0 0
9	0.93	0.02	0.9	0.02	0 0
10	1	0.01	0.99	0.01	0 0
11	1.04	0	1.03	0	0 0
12	0.35	-0.21	0.63	-0.17	-0.05 0.05
13	-0.5	-0.09	-0.35	-0.11	0 0
14	-0.89	-0.04	-0.82	-0.05	0 0
15	-1.06	-0.02	-1.03	-0.02	0 0
16	-0.41	0.2	-0.69	0.16	0.05 0.05
17	0.43	0.09	0.28	0.11	0 0
18	0.81	0.04	0.74	0.05	0 0
19	0.98	0.02	0.95	0.02	0 0
20	1.06	0.01	1.04	0.01	0 0

DAMPING1VELOCITY6SAMPLING PERIOD1

NO	X11	X21	SX1	SX2	K
1	1	0	1	0	0
2	0.34	-0.19	0.6	-0.16	-0.05 0.05
3	-0.38	-0.07	-0.26	-0.09	0 0
4	-0.65	-0.03	-0.6	-0.03	0 0
5	-0.74	-0.01	-0.73	-0.01	0 0
6	-0.78	0	-0.77	0	0 0
7	-0.79	0	-0.79	0	0 0
8	-0.8	0	-0.8	0	0 0
9	-0.8	0	-0.8	0	0 0
10	-0.8	0	-0.8	0	0 0
11	-0.8	0	-0.8	0	0 0
12	-0.8	0	-0.8	0	0 0
13	-0.8	0	-0.8	0	0 0
14	-0.8	0	-0.8	0	0 0
15	-0.8	0	-0.8	0	0 0
16	-0.8	0	-0.8	0	0 0
17	-0.8	0	-0.8	0	0 0
18	-0.8	0	-0.8	0	0 0
19	-0.8	0	-0.8	0	0 0
20	-0.8	0	-0.8	0	0 0

DAMPING0.3VELOCITY7SAMPLING PERIOD1

NO	X11	X21	SX1	SX2	K			
1	1	0	1	0	0	0		
2	0.05	-0.24	0.43	-0.2	-0.05	0.05		
3	-1.12	-0.11	-0.31	-0.13	0	0		
4	-1.64	-0.05	-1.54	-0.06	0	0		
5	-0.86	0.23	-1.22	0.18	0.05	0.03		
6	0.94	0.27	0.46	0.27	0.05	0.04		
7	2.26	0.12	2.02	0.15	0	0		
8	1.78	-0.21	2.11	-0.15	-0.05	0.02		
9	-0.06	-0.3	0.46	-0.23	-0.05	0.02		
10	-1.5	-0.13	-1.24	-0.16	0	0		
11	-1	0.23	-1.35	0.16	0.05	0.04		
12	0.82	0.28	0.34	0.27	0.05	0.04		
13	2.13	0.13	1.94	0.15	0	0		
14	1.72	-0.21	2.04	-0.15	-0.05	0.03		
15	-0.12	-0.3	0.39	-0.28	-0.05	0.02		
16	-1.56	-0.13	-1.3	-0.16	0	0		
17	-1.06	0.23	-1.41	0.16	0.05	0.04		
18	0.76	0.28	0.27	0.27	0.05	0.04		
19	2.13	0.13	1.89	0.16	0	0		
20	1.67	-0.22	1.93	-0.15	-0.05	0.03		

DAMPING1VELOCITY7SAMPLING PERIOD1

NO	X11	X21	SX1	SX2	K			
1	1	0	1	0	0	0		
2	0.1	-0.22	0.46	-0.16	-0.05	0.05		
3	-0.88	-0.08	-0.72	-0.1	0	0		
4	-1.24	-0.03	-1.13	-0.04	0	0		
5	-0.43	0.22	-0.76	0.13	0.05	0.04		
6	0.55	0.08	0.39	0.1	0	0		
7	0.91	0.03	0.85	0.04	0	0		
8	1.05	0.01	1.02	0.01	0	0		
9	0.17	-0.22	0.53	-0.18	-0.05	0.05		
10	-0.81	-0.08	-0.65	-0.1	0	0		
11	-1.17	-0.03	-1.11	-0.04	0	0		
12	-0.35	0.22	-0.71	0.16	0.05	0.05		
13	0.63	0.08	0.47	0.1	0	0		
14	0.39	0.03	0.33	0.04	0	0		
15	1.13	0.01	1.1	0.01	0	0		
16	0.26	-0.22	0.61	-0.16	-0.05	0.05		
17	-0.76	-0.08	-0.56	-0.1	0	0		
18	-1.03	-0.03	-1.03	-0.04	0	0		
19	-0.26	0.22	-0.62	0.18	0.05	0.05		
20	0.72	0.03	0.56	0.11	0	0		

24 PINGO.37 VELOCITY8SAMPLING PERIOD1

NO X11 X21 SX1 SX2 K

1	1	0	1	0	0	0
2	-0.25	-0.26	0.26	-0.23	-0.05	0.05
3	-1.76	-0.12	-1.49	-0.15	0	0
4	-0.37	0.27	-1.44	0.2	0.05	0.03
5	1.36	0.31	0.75	0.3	0.05	0.03
6	3.05	0.14	2.74	0.17	0	0
7	2.42	-0.24	2.34	-0.13	-0.05	0.02
8	0.01	-0.34	0.69	-0.33	-0.05	0.02
9	-1.86	-0.15	-1.54	-0.19	0	0
10	-1.21	0.27	-1.67	0.13	0.05	0.03
11	1.17	0.32	0.54	0.31	0.05	0.03
12	2.32	0.14	2.61	0.18	0	0
13	2.32	-0.24	2.74	-0.17	-0.05	0.02
14	-0.02	-0.34	0.53	-0.33	-0.05	0.02
15	-1.37	-0.15	-1.63	-0.19	0	0
16	-1.31	0.26	-1.77	0.19	0.05	0.03
17	1.06	0.32	0.43	0.31	0.05	0.03
18	2.84	0.15	2.52	0.18	0	0
19	2.24	-0.25	2.66	-0.17	-0.05	0.02
20	-0.16	-0.34	0.51	-0.32	-0.05	0.02

DAMPING1VELOCITY8SAMPLING PERIOD1

NO X11 X21 SX1 SX2 K

1	1	0	1	0	0	0
2	-0.18	-0.25	0.23	-0.21	-0.05	0.05
3	-1.46	-0.09	-1.24	-0.12	0	0
4	-0.55	0.26	-1.02	0.2	0.05	0.04
5	1.41	0.23	0.34	0.24	0.05	0.05
6	2.59	0.09	2.39	0.11	0	0
7	1.75	-0.24	2.18	-0.19	-0.05	0.02
8	-0.42	-0.29	0.16	-0.28	-0.05	0.02
9	-1.89	-0.11	-1.65	-0.14	0	0
10	-1.09	0.25	-1.53	0.13	0.05	0.03
11	1.02	0.27	0.43	0.27	0.05	0.03
12	2.39	0.1	2.16	0.13	0	0
13	1.59	-0.24	2.03	-0.13	-0.05	0.02
14	-0.56	-0.29	0.01	-0.28	-0.05	0.02
15	-2.02	-0.11	-1.78	-0.14	0	0
16	-1.22	0.25	-1.66	0.19	0.05	0.03
17	0.9	0.28	0.35	0.27	0.05	0.03
18	2.3	0.1	2.07	0.13	0	0
19	1.51	-0.24	1.34	-0.19	-0.05	0.02
20	-0.64	-0.29	-0.07	-0.28	-0.05	0.03

GRL

DAMPING 0.3 VELOCITY 5 SAMPLING PERIOD 0.8

NO	X11	X21	SX1	SX2	K			
1	1	0	1	0	0	0		
2	0.67	-0.15	0.84	-0.11	-0.05	0.05		
3	0.24	-0.08	0.34	-0.1	0	0		
4	0.01	-0.04	0.06	-0.05	0	0		
5	-0.11	-0.02	-0.08	-0.03	0	0		
6	-0.18	-0.01	-0.16	-0.01	0	0		
7	-0.21	-0.01	-0.2	-0.01	0	0		
8	-0.23	0	-0.23	0	0	0		
9	-0.24	0	-0.24	0	0	0		
10	-0.24	0	-0.24	0	0	0		
11	-0.25	0	-0.25	0	0	0		
12	-0.25	0	-0.25	0	0	0		
13	-0.25	0	-0.25	0	0	0		
14	-0.25	0	-0.25	0	0	0		
15	-0.25	0	-0.25	0	0	0		
16	-0.25	0	-0.25	0	0	0		
17	-0.25	0	-0.25	0	0	0		
18	-0.25	0	-0.25	0	0	0		
19	-0.25	0	-0.25	0	0	0		
20	-0.25	0	-0.25	0	0	0		

DAMPING 1 VELOCITY 5 SAMPLING PERIOD 0.8

NO	X11	X21	SX1	SX2	K			
1	1	0	1	0	0	0		
2	0.63	-0.14	0.84	-0.11	-0.05	0.05		
3	0.31	-0.06	0.4	-0.03	0	0		
4	0.14	-0.03	0.13	-0.04	0	0		
5	0.06	-0.01	0.03	-0.02	0	0		
6	0.03	-0.01	0.04	-0.01	0	0		
7	0.01	0	0.02	0	0	0		
8	0.01	0	0.01	0	0	0		
9	0	0	0	0	0	0		
10	0	0	0	0	0	0		
11	0	0	0	0	0	0		
12	0	0	0	0	0	0		
13	0	0	0	0	0	0		
14	0	0	0	0	0	0		
15	0	0	0	0	0	0		
16	0	0	0	0	0	0		
17	0	0	0	0	0	0		
18	0	0	0	0	0	0		
19	0	0	0	0	0	0		
20	0	0	0	0	0	0		

DAMPING0.8VELOCITYSAMPLING PERIOD0.8

NO	X11	X21	SX1	SX2	K			
1	1	0	1	0	0	0		
2	0.53	-0.18	0.76	-0.13	-0.05	0.05		
3	-0.1	-0.09	0.06	-0.11	0	0		
4	-0.43	-0.05	0.35	-0.06	0	0		
5	-0.61	-0.03	0.56	-0.03	0	0		
6	-0.7	-0.01	0.67	-0.02	0	0		
7	-0.75	-0.01	0.73	-0.01	0	0		
8	-0.77	0	0.77	0	0	0		
9	-0.78	0	0.78	0	0	0		
10	-0.79	0	0.79	0	0	0		
11	-0.8	0	0.79	0	0	0		
12	-0.8	0	0.8	0	0	0		
13	-0.8	0	0.8	0	0	0		
14	-0.8	0	0.8	0	0	0		
15	-0.8	0	0.8	0	0	0		
16	-0.8	0	0.8	0	0	0		
17	-0.8	0	0.8	0	0	0		
18	-0.8	0	0.8	0	0	0		
19	-0.8	0	0.8	0	0	0		
20	-0.8	0	0.8	0	0	0		

DAMPING1VELOCITYSAMPLING PERIOD0.8

NO	X11	X21	SX1	SX2	K			
1	1	0	1	0	0	0		
2	0.55	-0.17	0.77	-0.13	-0.05	0.05		
3	0.01	-0.07	0.13	-0.1	0	0		
4	-0.24	-0.03	0.18	-0.04	0	0		
5	-0.35	-0.01	0.32	-0.02	0	0		
6	-0.4	-0.01	0.39	-0.01	0	0		
7	-0.42	0	0.42	0	0	0		
8	-0.43	0	0.43	0	0	0		
9	-0.44	0	0.44	0	0	0		
10	-0.44	0	0.44	0	0	0		
11	-0.44	0	0.44	0	0	0		
12	-0.44	0	0.44	0	0	0		
13	-0.44	0	0.44	0	0	0		
14	-0.44	0	0.44	0	0	0		
15	-0.44	0	0.44	0	0	0		
16	-0.44	0	0.44	0	0	0		
17	-0.44	0	0.44	0	0	0		
18	-0.44	0	0.44	0	0	0		
19	-0.44	0	0.44	0	0	0		
20	-0.44	0	0.44	0	0	0		

DAMPING0.8VELOCITY7SAMPLING PERIOD0.8

NO X11 X21 SX1 SX2 K

1	1	0	1	0	0	0
2	0.36	-0.21	0.68	-0.15	-0.05	0.05
3	-0.5	-0.11	-0.28	-0.13	0	0
4	-0.35	-0.06	-0.84	-0.07	0	0
5	-1.18	-0.03	-1.13	-0.04	0	0
6	-0.64	0.2	-0.94	0.14	0.05	0.04
7	0.2	0.11	-0.01	0.13	0	0
8	0.64	0.06	0.53	0.07	0	0
9	0.87	0.03	0.81	0.04	0	0
10	0.39	0.02	0.36	0.02	0	0
11	1.06	0.01	1.04	0.01	0	0
12	0.44	-0.21	0.75	-0.15	-0.05	0.05
13	-0.41	-0.11	-0.2	-0.13	0	0
14	-0.86	-0.06	-0.75	-0.07	0	0
15	-1.1	-0.03	-1.04	-0.04	0	0
16	-0.54	0.2	-0.85	0.15	0.05	0.05
17	0.29	0.11	0.09	0.13	0	0
18	0.74	0.06	0.63	0.07	0	0
19	0.97	0.03	0.91	0.04	0	0
20	1.03	0.02	1.06	0.02	0	0

DAMPING1VELOCITY7SAMPLING PERIOD0.8

NO X11 X21 SX1 SX2 K

1	1	0	1	0	0	0
2	0.39	-0.19	0.69	-0.15	-0.05	0.05
3	-0.35	-0.09	-0.13	-0.11	0	0
4	-0.69	-0.04	-0.61	-0.05	0	0
5	-0.84	-0.02	-0.3	-0.02	0	0
6	-0.91	-0.01	-0.89	-0.01	0	0
7	-0.94	0	-0.93	0	0	0
8	-0.95	0	-0.95	0	0	0
9	-0.96	0	-0.95	0	0	0
10	-0.96	0	-0.96	0	0	0
11	-0.96	0	-0.96	0	0	0
12	-0.96	0	-0.96	0	0	0
13	-0.96	0	-0.96	0	0	0
14	-0.96	0	-0.96	0	0	0
15	-0.96	0	-0.96	0	0	0
16	-0.96	0	-0.96	0	0	0
17	-0.96	0	-0.96	0	0	0
18	-0.96	0	-0.96	0	0	0
19	-0.96	0	-0.96	0	0	0
20	-0.96	0	-0.96	0	0	0

DAMPING0.8VELOCITY8SAMPLING PERIOD0.3

NO X11 X21 SX1 SX2 K

1	1	0	1	0	0	0
2	0.16	-0.24	0.58	-0.16	-0.05	0.05
3	-0.95	-0.12	-0.68	-0.15	0	0
4	-1.54	-0.07	-1.4	-0.08	0	0
5	-0.93	0.23	-1.31	0.15	0.05	0.04
6	0.73	0.29	0.17	0.27	0.05	0.04
7	2.09	0.15	1.75	0.18	0	0
8	1.81	-0.2	2.13	-0.11	-0.05	0.03
9	0.14	-0.31	0.73	-0.28	-0.05	0.02
10	-1.31	-0.16	-0.95	-0.2	0	0
11	-2.08	-0.09	-1.89	-0.1	0	0
12	-1.56	0.21	-1.32	0.14	0.05	0.03
13	0.14	0.31	-0.45	0.28	0.05	0.03
14	1.58	0.16	1.22	0.2	0	0
15	1.26	-0.22	1.61	-0.13	-0.05	0.04
16	-0.43	-0.3	0.15	-0.28	-0.05	0.03
17	-1.36	-0.16	-1.51	-0.19	0	0
18	-1.58	0.21	-1.9	0.12	0.05	0.03
19	0.1	0.31	-0.49	0.23	0.05	0.03
20	1.54	0.16	1.18	0.2	0	0

DAMPING1VELOCITY8SAMPLING PERIOD0.3

NO X11 X21 SX1 SX2 K

1	1	0	1	0	0	0
2	0.2	-0.22	0.59	-0.17	-0.05	0.05
3	-0.77	-0.1	-0.54	-0.13	0	0
4	-1.2	-0.04	-1.1	-0.06	0	0
5	-0.53	0.22	-0.91	0.16	0.05	0.05
6	0.44	0.1	0.22	0.13	0	0
7	0.86	0.04	0.78	0.06	0	0
8	1.07	0.02	1.03	0.03	0	0
9	0.33	-0.22	0.72	-0.17	-0.05	0.05
10	-0.64	-0.1	-0.42	-0.13	0	0
11	-1.08	-0.04	-0.98	-0.06	0	0
12	-1.23	-0.02	-1.23	-0.03	0	0
13	-0.54	0.22	-0.92	0.16	0.05	0.04
14	0.43	0.1	0.2	0.13	0	0
15	0.86	0.04	0.76	0.06	0	0
16	1.06	0.02	1.01	0.03	0	0
17	0.31	-0.22	0.7	-0.17	-0.05	0.05
18	-0.66	-0.1	-0.44	-0.13	0	0
19	-1.1	-0.04	-1	-0.06	0	0
20	-1.3	-0.02	-1.25	-0.03	0	0

DAMPING0.8VELOCITY5SAAMPLING PERIOD0.6

NO X11 X21 SX1 SX2 K

1	1	0	1	0	0	0			
2	0.81	-	0.12	0.93	-	0.08	-	0.05	0.05
3	0.52	-	0.07	0.63	-	0.09	0	0	
4	0.35	-	0.05	0.41	-	0.06	0	0	
5	0.24	-	0.03	0.28	-	0.03	0	0	
6	0.17	-	0.02	0.2	-	0.02	0	0	
7	0.13	-	0.01	0.15	-	0.01	0	0	
8	0.1	-	0.01	0.11	-	0.01	0	0	
9	0.09	0	0.02	-	0.01	0	0		
10	0.08	0	0.06	0	0	0			
11	0.07	0	0.07	0	0	0			
12	0.07	0	0.07	0	0	0			
13	0.07	0	0.07	0	0	0			
14	0.06	0	0.07	0	0	0			
15	0.06	0	0.06	0	0	0			
16	0.06	0	0.06	0	0	0			
17	0.06	0	0.06	0	0	0			
18	0.06	0	0.06	0	0	0			
19	0.06	0	0.06	0	0	0			
20	0.06	0	0.06	0	0	0			

DAMPING1VELOCITY5SAAMPLING PERIOD0.6

NO X11 X21 SX1 SX2 K

1	1	0	1	0	0	0			
2	0.81	-	0.11	0.93	-	0.07	-	0.05	0.05
3	0.56	-	0.06	0.65	-	0.08	0	0	
4	0.42	-	0.03	0.47	-	0.04	0	0	
5	0.34	-	0.02	0.37	-	0.02	0	0	
6	0.3	-	0.01	0.32	-	0.01	0	0	
7	0.28	-	0.01	0.29	-	0.01	0	0	
8	0.27	0	0.27	0	0	0			
9	0.26	0	0.26	0	0	0			
10	0.25	0	0.26	0	0	0			
11	0.25	0	0.25	0	0	0			
12	0.25	0	0.25	0	0	0			
13	0.25	0	0.25	0	0	0			
14	0.25	0	0.25	0	0	0			
15	0.25	0	0.25	0	0	0			
16	0.25	0	0.25	0	0	0			
17	0.25	0	0.25	0	0	0			
18	0.25	0	0.25	0	0	0			
19	0.25	0	0.25	0	0	0			
20	0.25	0	0.25	0	0	0			

DAMPING VELOCITY SAMPLING PERIOD 0.6

NO X11 X21 SX1 SX2 K

1	1	0	1	0	0	0
2	0.72	-0.14	0.2	-0.09	-0.05	0.05
3	0.31	-0.09	0.46	-0.11	0	0
4	0.06	-0.05	0.15	-0.07	0	0
5	-0.1	-0.03	-0.04	-0.04	0	0
6	-0.19	-0.02	-0.16	-0.03	0	0
7	-0.25	-0.01	-0.23	-0.02	0	0
8	-0.29	-0.01	-0.28	-0.01	0	0
9	-0.31	0	-0.3	-0.01	0	0
10	-0.33	0	-0.32	0	0	0
11	-0.34	0	-0.33	0	0	0
12	-0.34	0	-0.34	0	0	0
13	-0.34	0	-0.34	0	0	0
14	-0.35	0	-0.35	0	0	0
15	-0.35	0	-0.35	0	0	0
16	-0.35	0	-0.35	0	0	0
17	-0.35	0	-0.35	0	0	0
18	-0.35	0	-0.35	0	0	0
19	-0.35	0	-0.35	0	0	0
20	-0.35	0	-0.35	0	0	0

DAMPING VELOCITY SAMPLING PERIOD 0.6

NO X11 X21 SX1 SX2 K

1	1	0	1	0	0	0
2	0.73	-0.14	0.9	-0.09	-0.05	0.05
3	0.37	-0.07	0.49	-0.1	0	0
4	0.16	-0.04	0.23	-0.05	0	0
5	0.05	-0.02	0.09	-0.03	0	0
6	-0.01	-0.01	0.01	-0.02	0	0
7	-0.04	-0.01	-0.03	-0.01	0	0
8	-0.06	0	-0.05	0	0	0
9	-0.07	0	-0.06	0	0	0
10	-0.07	0	-0.07	0	0	0
11	-0.08	0	-0.08	0	0	0
12	-0.08	0	-0.08	0	0	0
13	-0.08	0	-0.08	0	0	0
14	-0.08	0	-0.08	0	0	0
15	-0.08	0	-0.08	0	0	0
16	-0.08	0	-0.08	0	0	0
17	-0.08	0	-0.08	0	0	0
18	-0.08	0	-0.08	0	0	0
19	-0.08	0	-0.08	0	0	0
20	-0.08	0	-0.08	0	0	0

DAMPING VELOCITY SAMPLING PERIOD 0.6

NO X11 X21 SX1 SX2 K

1	1	0	1	0	0	0
2	0.62	-0.17	0.86	-0.11	-0.05	0.05
3	0.07	-0.1	0.27	-0.13	0	0
4	-0.28	-0.06	-0.16	-0.06	0	0
5	-0.43	-0.04	-0.42	-0.05	0	0
6	-0.62	-0.02	-0.58	-0.03	0	0
7	-0.71	-0.02	-0.68	-0.02	0	0
8	-0.76	-0.01	-0.74	-0.01	0	0
9	-0.79	-0.01	-0.78	-0.01	0	0
10	-0.81	0	-0.8	0	0	0
11	-0.82	0	-0.81	0	0	0
12	-0.83	0	-0.82	0	0	0
13	-0.83	0	-0.83	0	0	0
14	-0.83	0	-0.83	0	0	0
15	-0.83	0	-0.83	0	0	0
16	-0.84	0	-0.84	0	0	0
17	-0.84	0	-0.84	0	0	0
18	-0.84	0	-0.84	0	0	0
19	-0.84	0	-0.84	0	0	0
20	-0.84	0	-0.84	0	0	0

DAMPING VELOCITY SAMPLING PERIOD 0.6

NO X11 X21 SX1 SX2 K

1	1	0	1	0	0	0
2	0.64	-0.16	0.87	-0.1	-0.05	0.05
3	0.14	-0.09	0.31	-0.11	0	0
4	-0.14	-0.05	-0.04	-0.06	0	0
5	-0.29	-0.03	-0.24	-0.03	0	0
6	-0.37	-0.01	-0.34	-0.02	0	0
7	-0.41	-0.01	-0.4	-0.01	0	0
8	-0.44	0	-0.43	-0.01	0	0
9	-0.45	0	-0.45	0	0	0
10	-0.46	0	-0.46	0	0	0
11	-0.47	0	-0.46	0	0	0
12	-0.47	0	-0.47	0	0	0
13	-0.47	0	-0.47	0	0	0
14	-0.47	0	-0.47	0	0	0
15	-0.47	0	-0.47	0	0	0
16	-0.47	0	-0.47	0	0	0
17	-0.47	0	-0.47	0	0	0
18	-0.47	0	-0.47	0	0	0
19	-0.47	0	-0.47	0	0	0
20	-0.47	0	-0.47	0	0	0

DAMPING 0.3 VELOCITY & SAMPLING PERIOD 0.6

NO X11 X21 SX1 SX2 K

1	1	0	1	0	0	0
2	0.51	-0.19	0.82	-0.12	-0.05	0.05
3	-0.22	-0.12	0.04	-0.14	0	0
4	-0.67	-0.07	-0.51	-0.09	0	0
5	-0.95	-0.05	-0.85	-0.06	0	0
6	-1.12	-0.03	-1.06	-0.03	0	0
7	-0.7	0.18	-1	0.11	0.05	0.05
8	0	0.11	-0.25	0.14	0	0
9	0.43	0.07	0.28	0.09	0	0
10	0.7	0.04	0.61	0.05	0	0
11	0.87	0.03	0.81	0.03	0	0
12	0.97	0.02	0.94	0.02	0	0
13	1.04	0.01	1.01	0.01	0	0
14	0.57	-0.19	0.86	-0.12	-0.05	0.05
15	-0.15	-0.12	0.11	-0.14	0	0
16	-0.59	-0.07	-0.43	-0.09	0	0
17	-0.87	-0.04	-0.77	-0.05	0	0
18	-1.04	-0.03	-0.98	-0.03	0	0
19	-1.14	-0.02	-1.11	-0.02	0	0
20	-0.7	0.19	-1	0.11	0.05	0.05

DAMPING 1 VELOCITY & SAMPLING PERIOD 0.6

NO X11 X21 SX1 SX2 K

1	1	0	1	0	0	0
2	0.52	-0.18	0.82	-0.12	-0.05	0.05
3	-0.13	-0.1	0.1	-0.13	0	0
4	-0.49	-0.05	-0.36	-0.07	0	0
5	-0.68	-0.03	-0.61	-0.04	0	0
6	-0.79	-0.02	-0.75	-0.02	0	0
7	-0.85	-0.01	-0.83	-0.01	0	0
8	-0.88	0	-0.87	-0.01	0	0
9	-0.9	0	-0.89	0	0	0
10	-0.91	0	-0.9	0	0	0
11	-0.91	0	-0.91	0	0	0
12	-0.92	0	-0.92	0	0	0
13	-0.92	0	-0.92	0	0	0
14	-0.92	0	-0.92	0	0	0
15	-0.92	0	-0.92	0	0	0
16	-0.92	0	-0.92	0	0	0
17	-0.92	0	-0.92	0	0	0
18	-0.92	0	-0.92	0	0	0
19	-0.92	0	-0.92	0	0	0
20	-0.92	0	-0.92	0	0	0

DAMPING 0.8 VELOCITY 5% SAMPLING PERIOD 0.4

NO X11 X21 SX1 SX2 K

1	1	0	1	0	0	0			
2	0.91	-0.09	0.99	-0.04	-0.05	0.05			
3	0.76	-0.06	0.85	-0.08	0	0			
4	0.56	-0.05	0.72	-0.06	0	0			
5	0.58	-0.03	0.63	-0.04	0	0			
6	0.52	-0.02	0.56	-0.03	0	0			
7	0.48	-0.02	0.51	-0.02	0	0			
8	0.45	-0.01	0.47	-0.02	0	0			
9	0.43	-0.01	0.44	-0.01	0	0			
10	0.42	-0.01	0.43	-0.01	0	0			
11	0.41	0	0.41	0.01	0	0			
12	0.4	0	0.4	0	0	0			
13	0.39	0	0.39	0	0	0			
14	0.39	0	0.39	0	0	0			
15	0.38	0	0.39	0	0	0			
16	0.38	0	0.38	0	0	0			
17	0.38	0	0.38	0	0	0			
18	0.38	0	0.38	0	0	0			
19	0.38	0	0.38	0	0	0			
20	0.38	0	0.38	0	0	0			

DAMPING 1 VELOCITY 5% SAMPLING PERIOD 0.4

NO X11 X21 SX1 SX2 K

1	1	0	1	0	0	0			
2	0.91	-0.08	0.99	-0.03	-0.05	0.05			
3	0.73	-0.06	0.85	-0.07	0	0			
4	0.69	-0.04	0.74	-0.05	0	0			
5	0.62	-0.02	0.66	-0.03	0	0			
6	0.58	-0.02	0.61	-0.02	0	0			
7	0.56	-0.01	0.57	-0.01	0	0			
8	0.54	-0.01	0.55	-0.01	0	0			
9	0.53	-0.01	0.53	-0.01	0	0			
10	0.52	0	0.52	0	0	0			
11	0.51	0	0.51	0	0	0			
12	0.51	0	0.51	0	0	0			
13	0.51	0	0.51	0	0	0			
14	0.5	0	0.5	0	0	0			
15	0.5	0	0.5	0	0	0			
16	0.5	0	0.5	0	0	0			
17	0.5	0	0.5	0	0	0			
18	0.5	0	0.5	0	0	0			
19	0.5	0	0.5	0	0	0			
20	0.5	0	0.5	0	0	0			

DAMPING 0.8 VELOCITY SAMPLING PERIOD 0.4

NO	X11	X21	EX1	EX2	K			
1	1	0	1	0	0	0		
2	0.87	-0.1	0.38	-0.04	-0.05	0.05		
3	0.66	-0.07	0.78	-0.09	0	0		
4	0.51	-0.05	0.6	-0.07	0	0		
5	0.39	-0.04	0.46	-0.05	0	0		
6	0.31	-0.03	0.36	-0.03	0	0		
7	0.26	-0.02	0.23	-0.03	0	0		
8	0.21	-0.02	0.24	-0.02	0	0		
9	0.18	-0.01	0.2	-0.01	0	0		
10	0.16	-0.01	0.17	-0.01	0	0		
11	0.14	-0.01	0.15	-0.01	0	0		
12	0.13	0	0.14	-0.01	0	0		
13	0.12	0	0.13	0	0	0		
14	0.12	0	0.12	0	0	0		
15	0.11	0	0.11	0	0	0		
16	0.11	0	0.11	0	0	0		
17	0.11	0	0.11	0	0	0		
18	0.1	0	0.11	0	0	0		
19	0.1	0	0.1	0	0	0		
20	0.1	0	0.1	0	0	0		

DAMPING 1 VELOCITY SAMPLING PERIOD 0.4

NO	X11	X21	EX1	EX2	K			
1	1	0	1	0	0	0		
2	0.87	-0.1	0.38	-0.04	-0.05	0.05		
3	0.66	-0.07	0.78	-0.09	0	0		
4	0.55	-0.04	0.62	-0.06	0	0		
5	0.46	-0.03	0.51	-0.04	0	0		
6	0.4	-0.02	0.43	-0.03	0	0		
7	0.36	-0.01	0.33	-0.02	0	0		
8	0.33	-0.01	0.35	-0.01	0	0		
9	0.32	-0.01	0.33	-0.01	0	0		
10	0.3	0	0.31	-0.01	0	0		
11	0.3	0	0.3	0	0	0		
12	0.29	0	0.29	0	0	0		
13	0.29	0	0.29	0	0	0		
14	0.28	0	0.22	0	0	0		
15	0.28	0	0.28	0	0	0		
16	0.28	0	0.28	0	0	0		
17	0.28	0	0.28	0	0	0		
18	0.28	0	0.28	0	0	0		
19	0.28	0	0.28	0	0	0		
20	0.28	0	0.28	0	0	0		

DAMPING0.8VELOCITY7SAMPLING PERIOD0.4

NO X11 X21 SX1 SX2 K

1	1	0	1	0	0	0
2	0.82	-0.12	0.97	-0.05	-0.05	0.05
3	0.54	-0.09	0.7	-0.11	0	0
4	0.33	-0.06	0.45	-0.08	0	0
5	0.16	-0.05	0.27	-0.06	0	0
6	0.07	-0.03	0.13	-0.04	0	0
7	-0.01	-0.02	0.03	-0.03	0	0
8	-0.07	-0.02	0.04	-0.02	0	0
9	-0.11	-0.01	0.09	-0.02	0	0
10	-0.14	-0.01	0.13	-0.01	0	0
11	-0.17	-0.01	0.15	-0.01	0	0
12	-0.18	0	0.17	-0.01	0	0
13	-0.19	0	0.19	0	0	0
14	-0.2	0	0.2	0	0	0
15	-0.21	0	0.21	0	0	0
16	-0.21	0	0.21	0	0	0
17	-0.22	0	0.21	0	0	0
18	-0.22	0	0.22	0	0	0
19	-0.22	0	0.22	0	0	0
20	-0.22	0	0.22	0	0	0

DAMPING1VELOCITY7SAMPLING PERIOD0.4

NO X11 X21 SX1 SX2 K

1	1	0	1	0	0	0
2	0.83	-0.12	0.97	-0.05	-0.05	0.05
3	0.56	-0.08	0.72	-0.1	0	0
4	0.33	-0.05	0.49	-0.07	0	0
5	0.26	-0.03	0.33	-0.04	0	0
6	0.18	-0.02	0.23	-0.03	0	0
7	0.13	-0.02	0.16	-0.02	0	0
8	0.09	-0.01	0.11	-0.01	0	0
9	0.07	-0.01	0.08	-0.01	0	0
10	0.05	0	0.06	-0.01	0	0
11	0.04	0	0.05	0	0	0
12	0.03	0	0.04	0	0	0
13	0.03	0	0.03	0	0	0
14	0.03	0	0.03	0	0	0
15	0.02	0	0.03	0	0	0
16	0.02	0	0.02	0	0	0
17	0.02	0	0.02	0	0	0
18	0.02	0	0.02	0	0	0
19	0.02	0	0.02	0	0	0
20	0.02	0	0.02	0	0	0

DAMPING0.8VELOCITY8SAMPLING PERIOD0.4

NO	X11	X21	SX1	SX2	K			
1	1	0	1	0	0	0		
2	0.77	-0.14	0.37	-0.06	-0.05	0.05		
3	0.39	-0.1	0.61	-0.12	0	0		
4	0.12	-0.07	0.28	-0.09	0	0		
5	-0.08	-0.05	0.04	-0.06	0	0		
6	-0.22	-0.04	-0.14	-0.05	0	0		
7	-0.32	-0.03	-0.26	-0.03	0	0		
8	-0.4	-0.02	-0.35	-0.02	0	0		
9	-0.45	-0.01	-0.42	-0.02	0	0		
10	-0.49	-0.01	-0.47	-0.01	0	0		
11	-0.52	-0.01	-0.51	-0.01	0	0		
12	-0.54	-0.01	-0.53	-0.01	0	0		
13	-0.56	0	-0.55	0	0	0		
14	-0.57	0	-0.56	0	0	0		
15	-0.58	0	-0.57	0	0	0		
16	-0.58	0	-0.58	0	0	0		
17	-0.59	0	-0.59	0	0	0		
18	-0.59	0	-0.59	0	0	0		
19	-0.59	0	-0.59	0	0	0		
20	-0.6	0	-0.59	0	0	0		

DAMPING1VELOCITY8SAMPLING PERIOD0.4

NO	X11	X21	SX1	SX2	K			
1	1	0	1	0	0	0		
2	0.77	-0.13	0.97	-0.06	-0.05	0.05		
3	0.43	-0.09	0.63	-0.11	0	0		
4	0.19	-0.06	0.33	-0.08	0	0		
5	0.04	-0.04	0.13	-0.05	0	0		
6	-0.07	-0.03	-0.01	-0.03	0	0		
7	-0.14	-0.02	-0.1	-0.02	0	0		
8	-0.13	-0.01	-0.16	-0.02	0	0		
9	-0.22	-0.01	-0.2	-0.01	0	0		
10	-0.24	-0.01	-0.22	-0.01	0	0		
11	-0.25	0	-0.24	0	0	0		
12	-0.26	0	-0.26	0	0	0		
13	-0.27	0	-0.26	0	0	0		
14	-0.27	0	-0.27	0	0	0		
15	-0.27	0	-0.27	0	0	0		
16	-0.28	0	-0.27	0	0	0		
17	-0.28	0	-0.28	0	0	0		
18	-0.28	0	-0.28	0	0	0		
19	-0.28	0	-0.28	0	0	0		
20	-0.28	0	-0.28	0	0	0		

END

APPENDIX 2 EXTENDED SPRING RADIUS

VGRL[]V

V GRL

```

[1]  T←0.6
[2]  STA:V←5
[3]  STR:A←0.8
[4]  STAT:K←0
[5]  ('DAMPING ';A;' VELOCITY ';V;' SAMPLING PERIOD ';T')
[6]  'NO X11 X21 SX1 SX2 K'
[7]  B←T-0.25
[8]  D←0
[9]  X1←1
[10] X2←0
[11] C←1
[12] K←0
[13] N←0
[14] SX1←0
[15] SX2←0
[16] P←(T-(1*-A×T)÷A)÷A
[17] G←(1*-A×T)÷A
[18] STAR:X11←(X1-X1×K×P×V*2)+(V×G×X2)+K×P×D×V*2
[19] X21←(-K×V×G×X1)+(X2*-A×T)+K×G×V×D
[20] G←(1*-I×B)÷A
[21] P←(P-G)÷A
[22] SX1←(X1-X1×K×P×V*2)+(V×G×X2)+K×P×D×V*2
[23] SX2←(-K×V×G×X1)+(X2*-P×A)+K×V×G×D
[24] C,2 RND X11,X21,SX1,SX2,N,K
[25] P←-SX1
[26] N←((~I∨J),(I←I≥2),J←P≤-1)/ 0 0.04 -0.04
[27] K←N:H
[28] X1←X11
[29] X2←X21
[30] C←C+1
[31] →(C≤30)/STAR
[32] ST:A←A+0.2
[33] →(A≤1)/STAT
[34] V←V+1
[35] →(V≤6)/STR
[36] T←T-0.2
[37] →(T≤0.25)/PD
[38] →STA
[39] PD:'END'

```

V


```

CPL
DAMPING 0.8 VELOCITY 7 SAMPLING PERIOD 1
NO X11 X21 SX1 SX2 K
1 1 0 1 0 0 0
2 0.24 -0.19 0.54 -0.16 -0.04 0.04
3 -0.09 -0.09 -0.52 -0.11 0 0
4 -1.11 -0.04 -1.03 -0.05 0 0
5 -0.48 0.19 -0.77 0.15 0.04 0.04
6 0.43 0.09 0.27 0.1 0 0
7 0.84 0.04 0.77 0.05 0 0
8 1.03 0.02 1 0.02 0 0
9 1.11 0.01 1.1 0.01 0 0
10 0.37 -0.19 0.68 -0.16 -0.04 0.04
11 -0.55 -0.09 -0.38 -0.11 0 0
12 -0.97 -0.04 -0.89 -0.05 0 0
13 -1.15 -0.02 -1.12 -0.02 0 0
14 -0.45 0.19 -0.75 0.15 0.04 0.04
15 0.47 0.09 0.3 0.1 0 0
16 0.88 0.04 0.81 0.05 0 0
17 1.07 0.02 1.03 0.02 0 0
18 0.36 -0.19 0.67 -0.15 -0.04 0.04
19 -0.56 -0.09 -0.39 -0.1 0 0
20 -0.97 -0.04 -0.9 -0.05 0 0
DAMPING 1 VELOCITY 7 SAMPLING PERIOD 1
NO X11 X21 SX1 SX2 K
1 1 0 1 0 0 0
2 0.28 -0.18 0.56 -0.15 -0.04 0.04
3 -0.5 -0.07 -0.37 -0.08 0 0
4 -0.79 -0.02 -0.74 -0.03 0 0
5 -0.9 -0.01 -0.88 -0.01 0 0
6 -0.94 0 -0.93 0 0 0
7 -0.95 0 -0.95 0 0 0
8 -0.96 0 -0.96 0 0 0
9 -0.96 0 -0.96 0 0 0
10 -0.96 0 -0.96 0 0 0
11 -0.96 0 -0.96 0 0 0
12 -0.96 0 -0.96 0 0 0
13 -0.96 0 -0.96 0 0 0
14 -0.96 0 -0.96 0 0 0
15 -0.96 0 -0.96 0 0 0
16 -0.96 0 -0.96 0 0 0
17 -0.96 0 -0.96 0 0 0
18 -0.96 0 -0.96 0 0 0
19 -0.96 0 -0.96 0 0 0
20 -0.96 0 -0.96 0 0 0

```


DAMPING 0.8 VELOCITY 8 SAMPLING PERIOD 1
 NO X11 X21 SX1 SX2 Y

1	1	0	1	0	0	0		
2	0	-0.22	0.4	-0.18	-0.04	0.04		
3	-1.21	-0.1	-0.99	-0.12	0	0		
4	-1.76	-0.04	-1.66	-0.25	0	0		
5	-0.34	0.21	-1.33	0.17	0.04	0.02		
6	0.94	0.25	0.44	0.25	0.04	0.03		
7	2.33	0.11	2.08	0.14	0	0		
8	1.84	-0.2	2.18	-0.14	-0.04	0.02		
9	-0.08	-0.27	0.45	-0.26	-0.04	0.02		
10	-1.59	-0.12	-1.32	-0.15	0	0		
11	-1.06	0.21	-1.43	0.15	0.04	0.03		
12	0.84	0.26	0.33	0.25	0.04	0.03		
13	2.26	0.12	2.01	0.14	0	0		
14	1.78	-0.2	2.12	-0.14	-0.04	0.02		
15	-0.14	-0.27	0.39	-0.26	-0.04	0.02		
16	-1.64	-0.12	-1.37	-0.15	0	0		
17	-1.13	0.21	-1.49	0.15	0.04	0.03		
18	0.78	0.26	0.27	0.25	0.04	0.03		
19	2.22	0.12	1.96	0.14	0	0		
20	1.73	-0.2	2.07	-0.14	-0.04	0.02		

DAMPING 1 VELOCITY 8 SAMPLING PERIOD 1
 NO X11 X21 SX1 SX2 Y

1	1	0	1	0	0	0		
2	0.06	-0.2	0.43	-0.17	-0.04	0.04		
3	-0.96	-0.07	-0.8	-0.1	0	0		
4	-1.34	-0.03	-1.28	-0.04	0	0		
5	-0.49	0.2	-0.86	0.16	0.04	0.03		
6	0.53	0.07	0.36	0.1	0	0		
7	0.91	0.03	0.84	0.04	0	0		
8	1.04	0.01	1.02	0.01	0	0		
9	0.13	-0.2	0.5	-0.17	-0.04	0.04		
10	-0.9	-0.07	-0.73	-0.1	0	0		
11	-1.27	-0.03	-1.21	-0.04	0	0		
12	-0.42	0.2	-0.79	0.16	0.04	0.03		
13	0.6	0.07	0.43	0.1	0	0		
14	0.98	0.03	0.92	0.04	0	0		
15	1.12	0.01	1.1	0.01	0	0		
16	0.21	-0.2	0.58	-0.17	-0.04	0.04		
17	-0.82	-0.07	-0.65	-0.1	0	0		
18	-1.19	-0.03	-1.13	-0.04	0	0		
19	-0.34	0.2	-0.71	0.17	0.04	0.04		
20	0.69	0.07	0.52	0.1	0	0		

DAMPING 0.8 VELOCITY 7 SAMPLING PERIOD 0.8

NO	X11	X21	CX1	CX2	K			
1	1	0	1	0	0	0		
2	0.49	-0.17	0.74	-0.12	-0.04	0.04		
3	-0.2	-0.09	-0.03	-0.11	0	0		
4	-0.56	-0.05	-0.47	-0.06	0	0		
5	-0.75	-0.02	-0.7	-0.03	0	0		
6	-0.85	-0.01	-0.82	-0.02	0	0		
7	-0.9	-0.01	-0.89	-0.01	0	0		
8	-0.93	0	-0.92	0	0	0		
9	-0.94	0	-0.94	0	0	0		
10	-0.95	0	-0.95	0	0	0		
11	-0.96	0	-0.95	0	0	0		
12	-0.96	0	-0.96	0	0	0		
13	-0.96	0	-0.96	0	0	0		
14	-0.96	0	-0.96	0	0	0		
15	-0.96	0	-0.96	0	0	0		
16	-0.96	0	-0.96	0	0	0		
17	-0.96	0	-0.96	0	0	0		
18	-0.96	0	-0.96	0	0	0		
19	-0.96	0	-0.96	0	0	0		
20	-0.96	0	-0.96	0	0	0		

DAMPING 1 VELOCITY 7 SAMPLING PERIOD 0.8

NO	X11	X21	CX1	CX2	K			
1	1	0	1	0	0	0		
2	0.51	-0.15	0.75	-0.12	-0.04	0.04		
3	-0.08	-0.07	0.05	-0.09	0	0		
4	-0.35	-0.03	-0.29	-0.04	0	0		
5	-0.47	-0.01	-0.44	-0.02	0	0		
6	-0.52	-0.01	-0.51	-0.01	0	0		
7	-0.55	0	-0.54	0	0	0		
8	-0.56	0	-0.56	0	0	0		
9	-0.56	0	-0.56	0	0	0		
10	-0.57	0	-0.57	0	0	0		
11	-0.57	0	-0.57	0	0	0		
12	-0.57	0	-0.57	0	0	0		
13	-0.57	0	-0.57	0	0	0		
14	-0.57	0	-0.57	0	0	0		
15	-0.57	0	-0.57	0	0	0		
16	-0.57	0	-0.57	0	0	0		
17	-0.57	0	-0.57	0	0	0		
18	-0.57	0	-0.57	0	0	0		
19	-0.57	0	-0.57	0	0	0		
20	-0.57	0	-0.57	0	0	0		

DAMPING 0.8 VELOCITY 8 SAMPLING PERIOD 0.6

NO X11 X21 SX1 SX2 K

1	1	0	1	0	0	0
2	0.33	-0.19	0.65	-0.14	-0.04	0.04
3	-0.56	-0.1	-0.34	-0.12	0	0
4	-1.03	-0.05	-0.92	-0.06	0	0
5	-1.28	-0.03	-1.22	-0.03	0	0
6	-0.71	0.18	-1.03	0.13	0.04	0.03
7	0.62	0.23	0.18	0.22	0.04	0.04
8	1.7	0.12	1.43	0.15	0	0
9	1.47	-0.16	1.73	-0.09	-0.04	0.03
10	0.14	-0.25	0.61	-0.22	-0.04	0.02
11	-1.02	-0.13	-0.73	-0.16	0	0
12	-1.64	-0.07	-1.48	-0.08	0	0
13	-1.22	0.17	-1.51	0.11	0.04	0.03
14	0.14	0.24	-0.34	0.23	0.04	0.03
15	1.29	0.13	1	0.16	0	0
16	1.04	-0.17	1.31	-0.1	-0.04	0.04
17	-0.32	-0.24	0.15	-0.22	-0.04	0.03
18	-1.46	-0.13	-1.18	-0.16	0	0
19	-1.23	0.17	-1.5	0.09	0.04	0.03
20	0.11	0.24	-0.36	0.22	0.04	0.03

DAMPING 1 VELOCITY 8 SAMPLING PERIOD 0.8

NO X11 X21 SX1 SX2 K

1	1	0	1	0	0	0
2	0.36	-0.18	0.68	-0.14	-0.04	0.04
3	-0.41	-0.08	-0.23	-0.1	0	0
4	-0.76	-0.04	-0.68	-0.05	0	0
5	-0.92	-0.02	-0.88	-0.02	0	0
6	-0.99	-0.01	-0.97	-0.01	0	0
7	-1.02	0	-1.01	0	0	0
8	-0.33	0.18	-0.71	0.13	0.04	0.04
9	0.38	0.08	0.2	0.1	0	0
10	0.73	0.04	0.65	0.05	0	0
11	0.89	0.02	0.35	0.02	0	0
12	0.96	0.01	0.94	0.01	0	0
13	0.99	0	0.98	0	0	0
14	1	0	1	0	0	0
15	1.01	0	1.01	0	0	0
16	0.37	-0.18	0.69	-0.14	-0.04	0.04
17	-0.4	-0.08	-0.22	-0.1	0	0
18	-0.75	-0.04	-0.67	-0.05	0	0
19	-0.91	-0.02	-0.87	-0.02	0	0
20	-0.98	-0.01	-0.96	-0.01	0	0

APPENDIX 3 EXPANDED BEAD ZONE

```

      VCRL[D]7
      V CRL
[1]   T←0.6
[2]   STA:V←5
[3]   STR:A←0.8
[4]   STAT:K←0
[5]   ('DAMPING ' ;A; ' VELOCITY ' ;V; ' SAMPLING PERIOD ' ;T)
[6]   'NO X11 X21 SX1 SX2 K'
[7]   B←T-0.25
[8]   D←0
[9]   X1←0
[10]  X2←0
[11]  C←1
[12]  K←0
[13]  N←0
[14]  SX1←0
[15]  SX2←0
[16]  P←(T-(1*-A×T):A):A
[17]  Q←(1*-A×T):A
[18]  STAR:X11←(X1-K×P×V×2)+(V×C×X2)+P×P×D×V×2
[19]  X21←(-K×V×C×X1)+(X2×*-A×T)+N×C×V×D
[20]  G←(1*-A×B):A
[21]  F←(B-G):A
[22]  SX1←(X1-K×F×V×2)+(V×C×F×2)+F×F×D×V×2
[23]  SX2←(-K×V×C×X1)+(X2×*-A×T)+N×V×C×D
[24]  C,2 RND X11,X21,SX1,SX2,N,K
[25]  N←-SX1
[26]  M←((~I∨J),(J←P≥2),J←N≤-2)/ 0 0.05 -0.05
[27]  N←N÷N
[28]  X1←X11
[29]  X2←X21
[30]  C←C+1
[31]  →(C≤30)/STAR
[32]  ST:A←A+0.2
[33]  →(A≤1)/STAT
[34]  V←V+1
[35]  →(V≤6)/STR
[36]  T←T-0.2
[37]  →(T≤0.25)/PD
[38]  →STA
[39]  PD:'END'

```


GEL

DAMPING 0.8 VELOCITY 7 SAMPLING PERIOD 1

NO X11 X21 SX1 SX2 K

1	2	0	2	0	0	0		
2	1.05	-0.24	1.43	-0.2	-0.05	0.03		
3	-0.12	-0.11	0.09	-0.13	0	0		
4	-0.64	-0.05	-0.54	-0.06	0	0		
5	-0.87	-0.02	-0.83	-0.03	0	0		
6	-0.98	-0.01	-0.96	-0.01	0	0		
7	-1.02	0	-1.02	-0.01	0	0		
8	-1.05	0	-1.04	0	0	0		
9	-1.05	0	-1.05	0	0	0		
10	-1.06	0	-1.06	0	0	0		
11	-1.06	0	-1.06	0	0	0		
12	-1.06	0	-1.06	0	0	0		
13	-1.06	0	-1.06	0	0	0		
14	-1.06	0	-1.06	0	0	0		
15	-1.06	0	-1.06	0	0	0		
16	-1.06	0	-1.06	0	0	0		
17	-1.06	0	-1.06	0	0	0		
18	-1.06	0	-1.06	0	0	0		
19	-1.06	0	-1.06	0	0	0		
20	-1.06	0	-1.06	0	0	0		

DAMPING 1 VELOCITY 7 SAMPLING PERIOD 1

NO X11 X21 SX1 SX2 K

1	2	0	2	0	0	0		
2	1.1	-0.22	1.46	-0.18	-0.05	0.03		
3	0.12	-0.08	0.28	-0.1	0	0		
4	-0.24	-0.03	-0.18	-0.04	0	0		
5	-0.37	-0.01	-0.35	-0.01	0	0		
6	-0.42	0	-0.41	-0.01	0	0		
7	-0.44	0	-0.44	0	0	0		
8	-0.45	0	-0.45	0	0	0		
9	-0.45	0	-0.45	0	0	0		
10	-0.45	0	-0.45	0	0	0		
11	-0.45	0	-0.45	0	0	0		
12	-0.45	0	-0.45	0	0	0		
13	-0.45	0	-0.45	0	0	0		
14	-0.45	0	-0.45	0	0	0		
15	-0.45	0	-0.45	0	0	0		
16	-0.45	0	-0.45	0	0	0		
17	-0.45	0	-0.45	0	0	0		
18	-0.45	0	-0.45	0	0	0		
19	-0.45	0	-0.45	0	0	0		
20	-0.45	0	-0.45	0	0	0		

DAMPING 0.8 VELOCITY 8 SAMPLING PERIOD 1

NO X11 X21 SX1 SX2 K

1	2	0	2	0	0	0
2	0.75	-0.28	1.26	-0.23	-0.05	0.03
3	-0.76	-0.12	-0.49	-0.15	0	0
4	-1.44	-0.06	-1.32	-0.07	0	0
5	-1.75	-0.02	-1.69	-0.03	0	0
6	-1.89	-0.01	-1.86	-0.01	0	0
7	-1.95	-0.01	-1.94	-0.01	0	0
8	-1.98	0	-1.97	0	0	0
9	-1.99	0	-1.99	0	0	0
10	-2	0	-1.99	0	0	0
11	-2	0	-2	0	0	0
12	-2	0	-2	0	0	0
13	-2	0	-2	0	0	0
14	-2	0	-2	0	0	0
15	-2	0	-2	0	0	0
16	-2	0	-2	0	0	0
17	-2	0	-2	0	0	0
18	-2	0	-2	0	0	0
19	-2	0	-2	0	0	0
20	-2	0	-2	0	0	0

DAMPING 1 VELOCITY 8 SAMPLING PERIOD 1

NO X11 X21 SX1 SX2 K

1	2	0	2	0	0	0
2	0.82	-0.25	1.29	-0.21	-0.05	0.03
3	-0.46	-0.09	-0.24	-0.12	0	0
4	-0.93	-0.03	-0.85	-0.04	0	0
5	-1.1	-0.01	-1.07	-0.02	0	0
6	-1.16	0	-1.15	-0.01	0	0
7	-1.19	0	-1.18	0	0	0
8	-1.19	0	-1.19	0	0	0
9	-1.2	0	-1.2	0	0	0
10	-1.2	0	-1.2	0	0	0
11	-1.2	0	-1.2	0	0	0
12	-1.2	0	-1.2	0	0	0
13	-1.2	0	-1.2	0	0	0
14	-1.2	0	-1.2	0	0	0
15	-1.2	0	-1.2	0	0	0
16	-1.2	0	-1.2	0	0	0
17	-1.2	0	-1.2	0	0	0
18	-1.2	0	-1.2	0	0	0
19	-1.2	0	-1.2	0	0	0
20	-1.2	0	-1.2	0	0	0

DAMPING 0.8 VELOCITY 7 SAMPLING PERIOD 0.8
 NO X11 X21 SX1 SX2 K

1	2	0	2	0	0	0
2	1.36	-0.21	1.68	-0.16	-0.05	0.03
3	0.5	-0.11	0.72	-0.13	0	0
4	0.05	-0.06	0.16	-0.07	0	0
5	-0.18	-0.03	-0.13	-0.04	0	0
6	-0.31	-0.02	-0.28	-0.02	0	0
7	-0.38	-0.01	-0.36	-0.01	0	0
8	-0.41	0	-0.4	-0.01	0	0
9	-0.43	0	-0.42	0	0	0
10	-0.44	0	-0.44	0	0	0
11	-0.44	0	-0.44	0	0	0
12	-0.45	0	-0.45	0	0	0
13	-0.45	0	-0.45	0	0	0
14	-0.45	0	-0.45	0	0	0
15	-0.45	0	-0.45	0	0	0
16	-0.45	0	-0.45	0	0	0
17	-0.45	0	-0.45	0	0	0
18	-0.45	0	-0.45	0	0	0
19	-0.45	0	-0.45	0	0	0
20	-0.45	0	-0.45	0	0	0

DAMPING 1 VELOCITY 7 SAMPLING PERIOD 0.8
 NO X11 X21 SX1 SX2 K

1	2	0	2	0	0	0
2	1.39	-0.19	1.69	-0.15	-0.05	0.03
3	0.65	-0.09	0.82	-0.11	0	0
4	0.31	-0.04	0.39	-0.05	0	0
5	0.16	-0.02	0.2	-0.02	0	0
6	0.09	-0.01	0.11	-0.01	0	0
7	0.06	0	0.07	0	0	0
8	0.05	0	0.05	0	0	0
9	0.04	0	0.05	0	0	0
10	0.04	0	0.04	0	0	0
11	0.04	0	0.04	0	0	0
12	0.04	0	0.04	0	0	0
13	0.04	0	0.04	0	0	0
14	0.04	0	0.04	0	0	0
15	0.04	0	0.04	0	0	0
16	0.04	0	0.04	0	0	0
17	0.04	0	0.04	0	0	0
18	0.04	0	0.04	0	0	0
19	0.04	0	0.04	0	0	0
20	0.04	0	0.04	0	0	0

DAMPING 0.8 VELOCITY 8 SAMPLING PERIOD 0.8

NO X11 X21 SX1 SX2 F

1	2	0	2	0	0	0
2	1.16	-0.24	1.58	-0.18	-0.05	0.03
3	0.05	-0.12	0.32	-0.15	0	0
4	-0.54	-0.07	-0.4	-0.08	0	0
5	-0.85	-0.03	-0.78	-0.04	0	0
6	-1.02	-0.02	-0.98	-0.02	0	0
7	-1.1	-0.01	-1.08	-0.01	0	0
8	-1.15	-0.01	-1.14	-0.01	0	0
9	-1.17	0	-1.17	0	0	0
10	-1.19	0	-1.18	0	0	0
11	-1.19	0	-1.19	0	0	0
12	-1.2	0	-1.2	0	0	0
13	-1.2	0	-1.2	0	0	0
14	-1.2	0	-1.2	0	0	0
15	-1.2	0	-1.2	0	0	0
16	-1.2	0	-1.2	0	0	0
17	-1.2	0	-1.2	0	0	0
18	-1.2	0	-1.2	0	0	0
19	-1.2	0	-1.2	0	0	0
20	-1.2	0	-1.2	0	0	0

DAMPING 1 VELOCITY 8 SAMPLING PERIOD 0.8

NO X11 X21 SX1 SX2 K

1	2	0	2	0	0	0
2	1.2	-0.22	1.59	-0.17	-0.05	0.03
3	0.23	-0.1	0.46	-0.13	0	0
4	-0.2	-0.04	-0.1	-0.06	0	0
5	-0.4	-0.02	-0.35	-0.03	0	0
6	-0.49	-0.01	-0.47	-0.01	0	0
7	-0.53	0	-0.52	-0.01	0	0
8	-0.55	0	-0.54	0	0	0
9	-0.55	0	-0.55	0	0	0
10	-0.56	0	-0.56	0	0	0
11	-0.56	0	-0.56	0	0	0
12	-0.56	0	-0.56	0	0	0
13	-0.56	0	-0.56	0	0	0
14	-0.56	0	-0.56	0	0	0
15	-0.56	0	-0.56	0	0	0
16	-0.56	0	-0.56	0	0	0
17	-0.56	0	-0.56	0	0	0
18	-0.56	0	-0.56	0	0	0
19	-0.56	0	-0.56	0	0	0
20	-0.56	0	-0.56	0	0	0

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